LECTURE NOTES ON

# ELECTRICAL MACHINES-I

## II B. Tech I semester

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# UNIT I

# Principles of Electromechanical Energy Conversion

Topics to cover:

1) Introduction
2) EMF in Electromechanical Systems
3) Force and Torque on a Conductor
4) Force and Torque Calculation from Energy and Co-energy
5) Model of Electromechanical Systems

## Introduction

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories:

(1) Transducers (for measurement and control)

These devices transform the signals of different forms. Examples are microphones, pickups, and speakers.

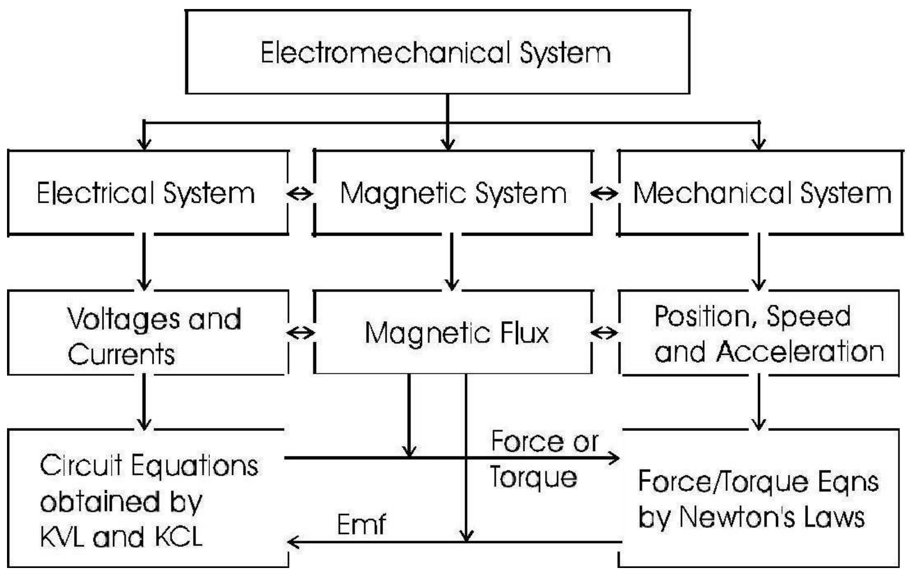
(2) Force producing devices (linear motion devices)

These type of devices produce forces mostly for linear motion drives, such as relays, solenoids (linear actuators), and electromagnets.

(3) Continuous energy conversion equipment

These devices operate in rotating mode. A device would be known as a generator if it convert mechanical energy into electrical energy, or as a motor if it does the other way around (from electrical to mechanical).

Since the permeability of ferromagnetic materials are much larger than the permittivity of dielectric materials, it is more advantageous to use electromagnetic field as the medium for electromechanical energy conversion. As illustrated in the following diagram, an electromechanical system consists of an electrical subsystem (electric circuits such as windings), a magnetic subsystem (magnetic field in the magnetic cores and airgaps), and a mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator and a rotor in a rotating electrical machine). Voltages and currents are used to describe the state of the electrical subsystem and they are governed by the basic circuital laws: Ohm's law, KCL and KVL. The state of the mechanical subsystem can be described in terms of positions, velocities, and accelerations, and is governed by the Newton's laws. The magnetic subsystem or magnetic field fits between the electrical and mechanical subsystems and acting as a "ferry" in energy transform and conversion. The field quantities such as magnetic flux, flux density, and field strength, are governed by the Maxwell's equations. When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movement of the moving part will could variation of the magnetic flux linking the electric circuit and induce an electromotive force (emf) in the circuit. The product of the torque and speed (the mechanical power) equals the active component of the product of the emf and current. Therefore, the electrical energy and the mechanical energy are inter-converted via the magnetic field.




Concept map of electromechanical system modeling

In this chapter, the methods for determining the induced emf in an electrical circuit and force/torque experienced by a movable part will be discussed. The general concept of electromechanical system modeling will also be illustrated by a singly excited rotating system.

## Induced emf in Electromechanical Systems

The diagram below shows a conductor of length l placed in a uniform magnetic field of flux density B. When the conductor moves at a speed v, the induced emf in the conductor can be determined by

$$e = l\mathbf{v} \times \mathbf{B}$$

The direction of the emf can be determined by the "right hand rule" for cross products. In a coil of N turns, the induced emf can be calculated by

$$e = -\frac{d\lambda}{dt}$$

where $\lambda$ is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement.

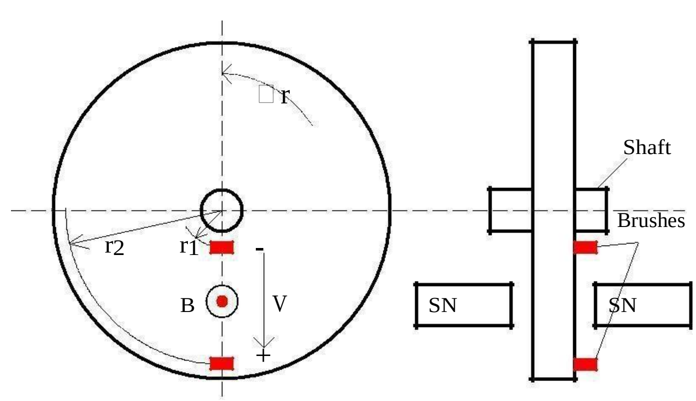
In practice, it would convenient if we treat the emf as a voltage. The above express can then be rewritten as

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self inductance is a function of the displacement x since there is a moving part in the system.

Example:

Calculate the open circuit voltage between the brushes on a Faraday's disc as shown schematically in the diagram below.



Solution:

Choose a small line segment of length dr at position r ($r_1 < r < r_2$) from the center of the disc between the brushes. The induced emf in this elemental length is then

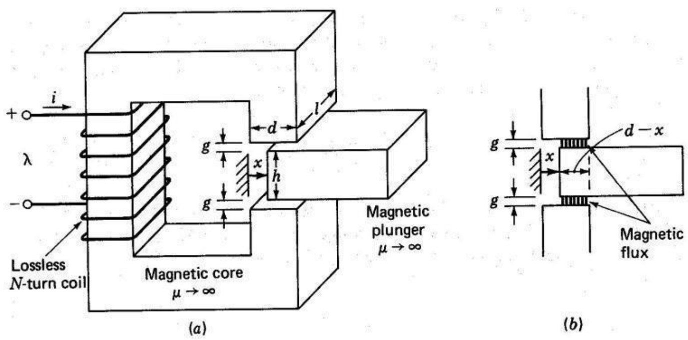
$$de = Bv\,dr = B\omega_r r\,dr$$

where $v = r\omega_r$. Therefore,

$$e = \int_{r_1}^{r_2} B\omega_r r\,dr = \omega_r B \left.\frac{r^2}{2}\right|_{r_1}^{r_2} = \omega_r B \frac{r_2^2 - r_1^2}{2}$$

Example:

Sketch L(x) and calculate the induced emf in the excitation coil for a linear actuator shown below.




A singly excited linear actuator

Solution:

$$L(x) = \frac{N^2}{R_g(x)}$$

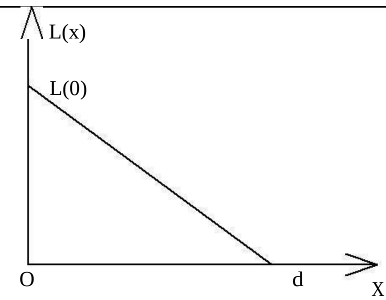
and

$$R_g(x) = \frac{2g}{\mu_o (d - x) l}$$

$$L(x) = \frac{\mu_o N^2 l (d - x)}{2g}$$

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

$$= L(x)\frac{di}{dt} - i\frac{\mu_o N^2 l}{2g} v$$




Inductance vs. displacement

If $i=I_{dc}$,

$$e = -I_{dc}\frac{\mu_o N^2 l}{2g}v$$

If $i=I_m \sin\omega t$,

$$e = \frac{\mu_o N^2 l}{2g}(d-x)I_m\omega\cos\omega t - vI_m\sin\omega t\,\frac{\mu_o N^2 l}{2g}$$

$$= I_m\frac{\mu_o N^2 l}{2g}\left[(d-x)\omega\cos\omega t - v\sin\omega t\right]$$

$$= I_m\frac{\mu_o N l^2}{2g}\sqrt{(d-x)^2\omega^2 + v^2}\,\cos\left(\omega t + \arctan\frac{v}{\omega(d-x)}\right)$$

## Force and Torque on a Current Carrying Conductor

The force on a moving particle of electric charge q in a magnetic field is given by the Lorentz's force law:

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

The force acting on a current carrying conductor can be directly derived from the equation as

$$\mathbf{F} = I\oint_C d\mathbf{l}\times\mathbf{B}$$

where C is the contour of the conductor. For a homogeneous conductor of length l carrying current I in a uniform magnetic field, the above expression can be reduced to

$$\mathbf{F} = I(\mathbf{l}\times\mathbf{B})$$

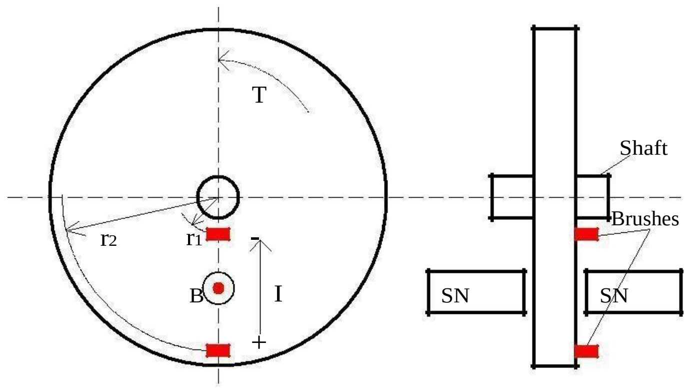
In a rotating system, the torque about an axis can be calculated by

$$\mathbf{T} = \mathbf{r}\times\mathbf{F}$$

where r is the radius vector from the axis towards the conductor.

Example:

Calculate the torque produced by the Faraday's disc if a dc current $I_{dc}$ flows from the positive terminal to the negative terminal as shown below.




Solution:

Choose a small segment of length dr at position r ($r_1 < r < r_2$) between the brushes. The force generated by this segment is

$$d\mathbf{F} = (I\,dr\,\mathbf{a}_r) \times (B\,\mathbf{a}_z) = -IB\,dr\,\mathbf{a}_\phi$$

where $\mathbf{a}_\phi$ is the unit vector in $\phi$ direction. The corresponding torque is

$$d\mathbf{T} = \mathbf{r} \times d\mathbf{F} = -IBr\,dr\,\mathbf{a}_z$$

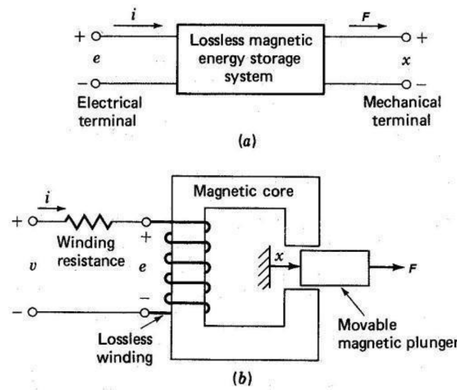
Therefore,

$$\mathbf{T} = \int d\mathbf{T} = \int_{r_1}^{r_2} -IBr\,dr\,\mathbf{a}_z = -IB\frac{r_2^2 - r_1^2}{2}\mathbf{a}_z$$

## Force and Torque Calculation from Energy and Co-energy

### A Singly Excited Linear Actuator

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant t, we record that the terminal voltage applied to the excitation winding is v, the excitation winding current i, the position of the movable plunger x, and the force acting on the plunger F with the reference direction chosen in the positive direction of the x axis, as shown in the diagram. After a time interval dt, we notice that the plunger has moved for a distance dx under the action of the force F. The mechanical done by the force acting on the plunger during this time interval is thus



A singly excited linear actuator

$$dW_m = Fdx$$

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dW_e = dW_f + dW_m = vidt - Ri^2 dt$$

Because $e = \frac{d\lambda}{dt} = v - Ri$

we can write

$$dW_f = dW_e - dW_m = eidt - Fdx = id\lambda - Fdx$$

From the above equation, we know that the energy stored in the magnetic field is a function of the flux linkage of the excitation winding and the position of the plunger. Mathematically, we can also write

$$dW_f(\lambda, x) = \frac{\partial W_f(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_f(\lambda, x)}{\partial x} dx$$

Therefore, by comparing the above two equations, we conclude

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad \text{and} \quad F = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

From the knowledge of electromagnetics, the energy stored in a magnetic field can be expressed as

$$W_f(\lambda, x) = \int_0^{\lambda} i(\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

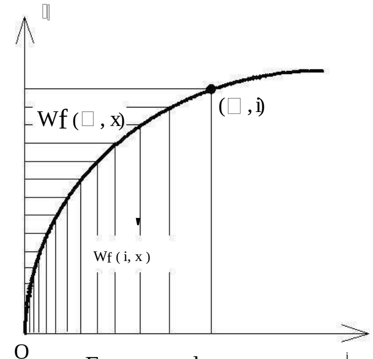
$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = \frac{1}{2}\left(\frac{\lambda}{L(x)}\right)^2\frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or $\lambda$-i curve. Mathematically, if we define the area underneath the magnetization curve as the co-energy (which does not exist physically), i.e.

$$W_f'(i, x) = i\lambda - W_f(\lambda, x)$$

we can obtain

$$dW_f'(i, x) = \lambda di + id\lambda - dW_f(\lambda, x)$$
$$= \lambda di + Fdx$$
$$= \frac{\partial W_f'(i, x)}{\partial i}di + \frac{\partial W_f'(i, x)}{\partial x}dx$$



Energy and co-energy

Therefore,

$$\lambda = \frac{\partial W_f'(i, x)}{\partial i}$$

and $$F = \frac{\partial W_f'(i, x)}{\partial x}$$

From the above diagram, the co-energy or the area underneath the magnetization curve can be calculated by

$$W_f'(i, x) = \int_0^i \lambda(i, x)\, di$$

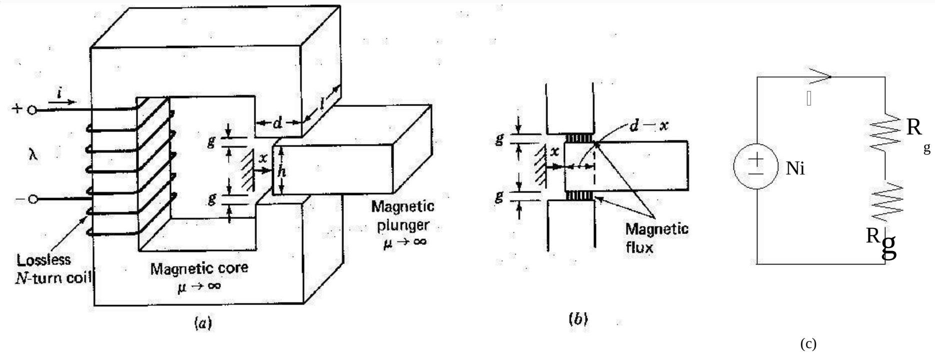
For a magnetically linear system, the above expression becomes

$$W_f'(i, x) = \frac{1}{2}i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f'(i, x)}{\partial x} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

| |
|---|
| Example:<br>Calculate the force acting on the plunger of a linear actuator discussed in this section. |




A singly excited linear actuator

Solution:

Assume the permeability of the magnetic core of the actuator is infinite, and hence the system can be treated as magnetically linear. From the equivalent magnetic circuit of the actuator shown in figure (c) above, one can readily find the self inductance of the excitation winding as

$$L(x) = \frac{N^2}{2R_g} = \frac{\mu_0 N^2 l(d-x)}{2g}$$

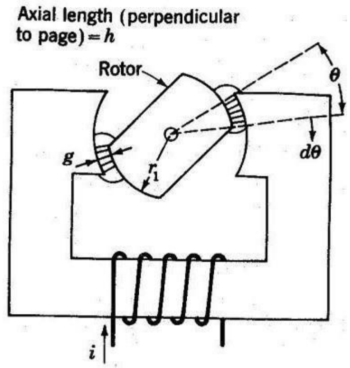
Therefore, the force acting on the plunger is

$$F = \frac{1}{2} i^2 \frac{dL(x)}{dx} = -\frac{\mu_0 l}{4g}(Ni)^2$$

The minus sign of the force indicates that the direction of the force is to reduce the displacement so as to reduce the reluctance of the air gaps. Since this force is caused by the variation of magnetic reluctance of the magnetic circuit, it is known as the reluctance force.

Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram below. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the co-energy against the angular displacement, as summarized in the following table.




A singly excited rotating actuator

Table: Torque in a singly excited rotating actuator

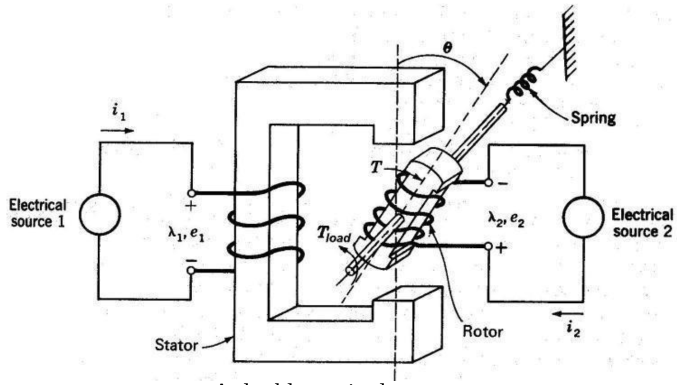
| Energy | Co-energy |
| --- | --- |
| In general, | |
| $dW_f = i\,d\lambda - T\,d\theta$ | $dW_f' = \lambda\,di + T\,d\theta$ |
| $W_f(\lambda,\theta) = \int_0^{\lambda} i(\lambda,\theta)\,d\lambda$ | $W_f'(i,\theta) = \int_0^{i} \lambda(i,\theta)\,di$ |
| $i = \dfrac{\partial W_f(\lambda,\theta)}{\partial \lambda}$ | $\lambda = \dfrac{\partial W_f'(i,\theta)}{\partial i}$ |
| $T = -\dfrac{\partial W_f(\lambda,\theta)}{\partial \theta}$ | $T = \dfrac{\partial W_f'(i,\theta)}{\partial \theta}$ |
| If the permeability is a constant, | |
| $W_f(\lambda,\theta) = \dfrac{1}{2}\dfrac{\lambda^2}{L(\theta)}$ | $W_f'(i,\theta) = \dfrac{1}{2} i^2 L(\theta)$ |
| $T = \dfrac{1}{2}\left(\dfrac{\lambda}{L(\theta)}\right)^2 \dfrac{dL(\theta)}{d\theta} = \dfrac{1}{2} i^2 \dfrac{dL(\theta)}{d\theta}$ | $T = \dfrac{1}{2} i^2 \dfrac{dL(\theta)}{d\theta}$ |

Doubly Excited Rotating Actuator

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and co-energy functions can be derived as following:

$$dW_f = dW_e - dW_m$$

where $dW_e = e_1 i_1 dt + e_2 i_2 dt$




A doubly excited actuator

$$e_1 = \frac{d\lambda_1}{dt} \qquad e_2 = \frac{d\lambda_2}{dt}$$

and
$$dW_m = T\,d\theta$$

Hence,

$$dW_f(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T d\theta$$
$$= \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} d\lambda_2 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} d\theta$$

and

$$dW_f'(i_1, i_2, \theta) = d\left(i_1\lambda_1 + i_2\lambda_2 - W_f(\lambda_1, \lambda_2, \theta)\right)$$
$$= \lambda_1 di_1 + \lambda_2 di_2 + T d\theta$$
$$= \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_2} di_2 + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} d\theta$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}$$

or
$$T = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

where $L_{12}=L_{21}$, $\Gamma_{11}=L_{22}/\Gamma$, $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Gamma$, $\Gamma_{22}=L_{11}/\Gamma$, and $\Gamma=L_{11}L_{22}-L_{12}^2$. The magnetic energy and co-energy can then be expressed as

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2$$

respectively, and it can be shown that they are equal.

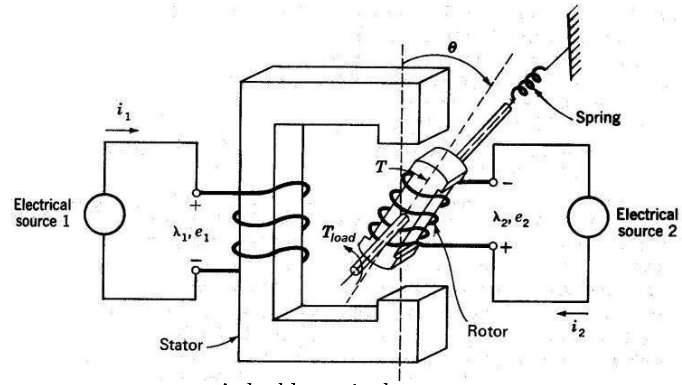
Therefore, the torque acting on the rotor can be calculated as

$$T = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} = \frac{1}{2}i_1^2\frac{dL_{11}}{d\theta} + \frac{1}{2}i_2^2\frac{dL_{22}}{d\theta} + i_1i_2\frac{dL_{12}}{d\theta}$$

Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that $dL_{11}/d\theta \neq 0$. Similarly, the second term on the right hand side of the above torque express is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

## Model of Electromechanical Systems

To illustrate the general principle for modeling of an electromechanical system, we still use the doubly excited rotating actuator discussed above as an example. For convenience, we plot it here again. As discussed in the introduction, the mathematical model of an electromechanical system consists of circuit equations for the electrical subsystem and force




A doubly excited actuator

or torque balance equations for the mechanical subsystem, whereas the interactions between the two subsystems via the magnetic field can be expressed in terms of the emf's and the electromagnetic force or torque. Thus, for the doubly excited rotating actuator, we can write

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + \frac{d(L_{11} i_1 + L_{12} i_2)}{dt}$$

$$= R_1 i_1 + L_{11}\frac{di_1}{dt} + i_1 \frac{dL_{11}}{d\theta}\frac{d\theta}{dt} + L_{12}\frac{di_2}{dt} + i_2 \frac{dL_{12}}{d\theta}\frac{d\theta}{dt}$$

$$= \left(R_1 + \omega_r \frac{dL_{11}}{d\theta}\right) i_1 + \omega_r \frac{dL_{12}}{d\theta} i_2 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$v_2 = R_2 i_2 + \frac{d\lambda_2}{dt} = R_2 i_2 + \frac{d(L_{21} i_1 + L_{22} i_2)}{dt}$$

$$= R_2 i_2 + L_{12}\frac{di_1}{dt} + i_1 \frac{dL_{12}}{d\theta}\frac{d\theta}{dt} + L_{22}\frac{di_2}{dt} + i_2 \frac{dL_{22}}{d\theta}\frac{d\theta}{dt}$$

$$= \omega_r \frac{dL_{12}}{d\theta} i_1 + \left(R_2 + \omega_r \frac{dL_{22}}{d\theta}\right) i_2 + L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

and
$$T - T_{load} = J\frac{d\omega_r}{dt}$$

where
$$\omega_r = \frac{d\theta}{dt}$$

is the angular speed of the rotor, $T_{load}$ the load torque, and J the inertia of the rotor and the mechanical load which is coupled to the rotor shaft.

The above equations are nonlinear differential equations which can only be solved numerically. In the format of state equations, the above equations can be rewritten as

$$\frac{di_1}{dt} = -\frac{1}{L_{11}}\left(R_1 + \frac{dL_{11}}{d\theta}\omega_r\right) i_1 - \frac{1}{L_{11}}\left(\frac{dL_{12}}{d\theta}\omega_r\right) i_2 - \frac{L_{12}}{L_{11}}\frac{di_2}{dt} + \frac{1}{L_{11}} v_1$$

$$\frac{di_2}{dt} = -\frac{1}{L_{22}}\left(\frac{dL_{12}}{d\theta}\omega_r\right) i_1 - \frac{1}{L_{22}}\left(R_2 + \frac{dL_{22}}{d\theta}\omega_r\right) i_2 - \frac{L_{12}}{L_{22}}\frac{di_1}{dt} + \frac{1}{L_{22}} v_2$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} T - \frac{1}{J} T_{load}$$

and

$$\frac{d\theta}{dt} = \omega_r$$

Together with the specified initial conditions (the state of the system at time zero in terms of the state variables):

$$I_1\big|_{t=0} = i_{10},\ i_2\big|_{t=0} = i_{20},\ \omega_r\big|_{t=0} = \omega_{r0},\ \text{and}\ \theta\big|_{t=0} = \theta_0,$$

the above state equations can be used to simulate the dynamic performance of the doubly excitedrotating actuator.

Following the same rule, we can derive the state equation model of anyelectromechanical systems.

# UNIT-II
# DC GENERATOR

The electrical machines deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form, this process is called electromechanical energy conversion. An electrical machine which converts mechanical energy into electrical energy is called an electric generator while an electrical machine which converts electrical energy into the mechanical energy is called an electric motor. A DC generator is built utilizing the basic principle that emf is induced in a conductor when it cuts magnetic lines of force. A DC motor works on the basic principle that a current carrying conductor placed in a magnetic field experiences a force.

## Working principle:

All the generators work on the principle of dynamically induced emf.

The change in flux associated with the conductor can exist only when there exists a relative motion between the conductor and the flux.

The relative motion can be achieved by rotating the conductor w.r.t flux or by rotating flux w.r.t conductor. So, a voltage gets generated in a conductor as long as there exists a relative motion between conductor and the flux. Such an induced emf which is due to physical movement of coil or conductor w.r.t flux or movement of flux w.r.t coil or conductor is called dynamically induced emf.

Whenever a conductor cuts magnetic flux, dynamically induced emf is produced in it according to Faraday's laws of Electromagnetic Induction.

This emf causes a current to flow if the conductor circuit is closed.

So, a generating action requires the following basic components to exist.

1. The conductor or a coil
2. Flux
3. Relative motion between the conductor and the flux.

In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as output, a number of conductors are connected together in a specific manner to form a winding. The winding is called armature winding of a dc machine and the part on which this winding is kept is called armature of the dc machine.

The magnetic field is produced by a current carrying winding which is called field winding.

The conductors placed on the armature are rotated with the help of some external device. Such an external device is called a prime mover.

The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc.

The purpose of the prime mover is to rotate the electrical conductor as required by Faraday's laws

The direction of induced emf can be obtained by using Flemings right hand rule.

The magnitude of induced emf = $e = BLV \sin\O = E_m \sin\O$

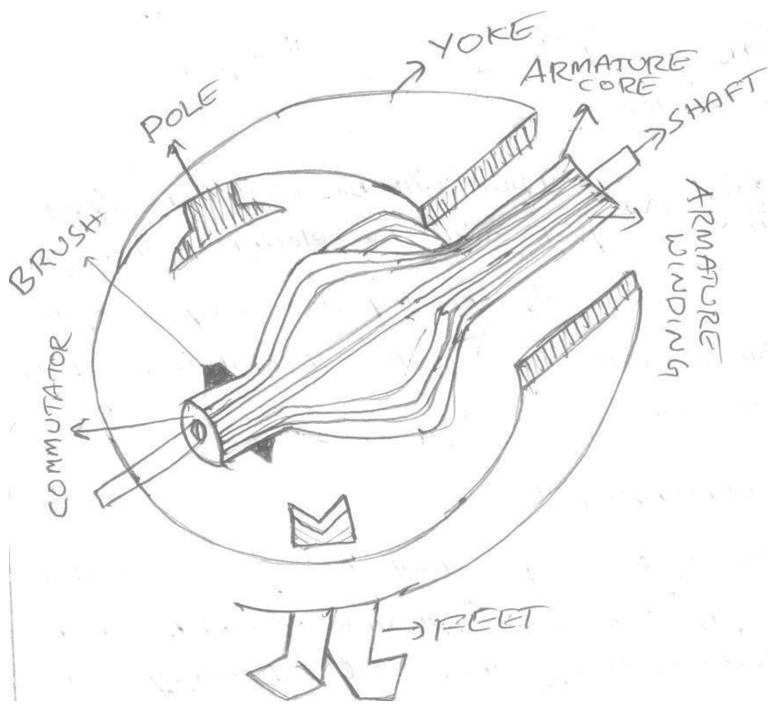
Nature of induced emf:

The nature of the induced emf for a conductor rotating in the magnetic field is alternating. As conductor rotates in a magnetic field, the voltage component at various positions is different. Hence the basic nature of induced emf in the armature winding in case of dc generator is alternating. To get dc output which is unidirectional, it is necessary to rectify the alternating induced emf. A device which is used in dc generator to convert alternating induced emf to unidirectional dc emf is called commutator.

Construction of DC machines :

A D. C. machine consists of two main parts

1. Stationary part: It is designed mainly for producing a magnetic flux.
2. Rotating part: It is called the armature, where mechanical energy is converted into electrical (electrical generate) or conversely electrical energy into mechanical (electric into)




Parts of a Dc Generator:

1) Yoke
2) Magnetic Poles
   a) Pole core
   b) Pole Shoe
3) Field Winding
4) Armature Core
5) Armature winding
6) Commutator
7) Brushes and Bearings

The stationary parts and rotating parts are separated from each other by an air gap. The stationary part of a D. C. machine consists of main poles, designed to create the magnetic flux, commutating poles interposed between the main poles and designed to ensure spark less operation of the brushes at the commutator and a frame / yoke. The armature is a cylindrical body rotating in the space between the poles and comprising a slotted armature core, a winding inserted in the armature core slots, a commutator and brush

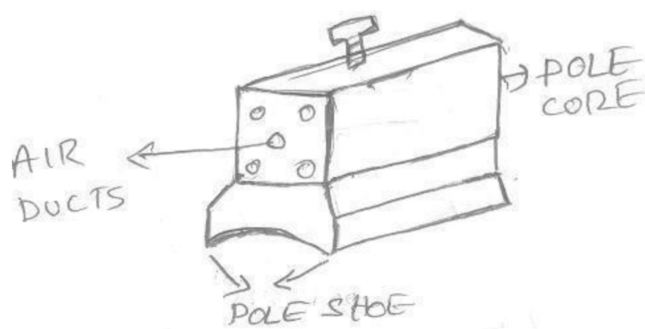
Yoke:

1. It saves the purpose of outermost cover of the dc machine so that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like $SO_2$, acidic fumes etc.
2. It provides mechanical support to the poles.
3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux.

Choice of material: To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is the cheapest. For large machines rolled steel or cast steel, is used which provides high permeability i.e., low reluctance and gives good mechanical strength.

Poles: Each pole is divided into two parts

a) pole core b) pole shoe




Functions:

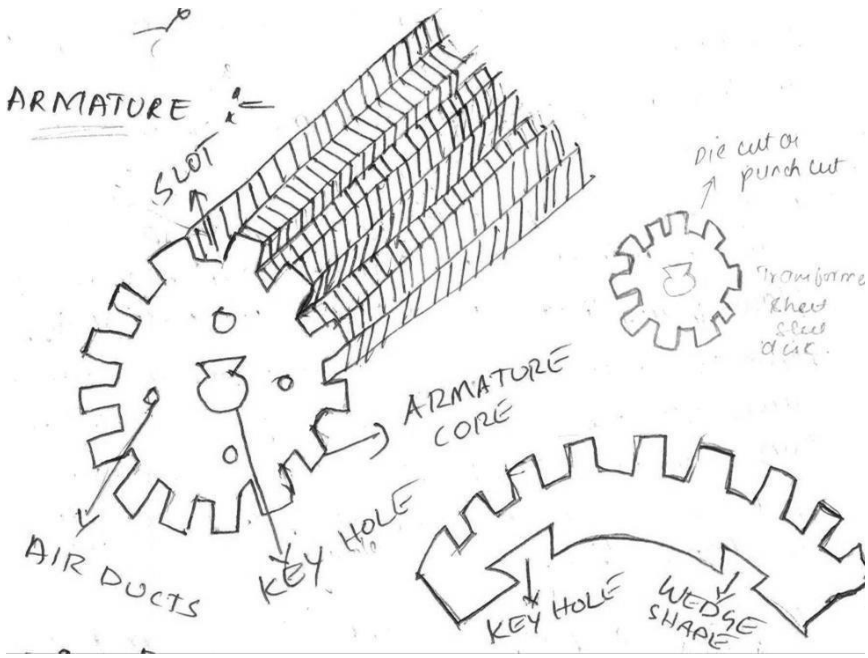
1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf. To achieve this, pole core has been given a particular shape.

Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to yoke.

Armature: It is further divided into two parts namely,

(1) Armature core (2) Armature winding.

Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.




Functions:

1. Armature core provides house for armature winding i.e., armature conductors.
2. To provide a path of low reluctance path to the flux it is made up of magnetic material like cast iron or cast steel.

Choice of material: As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

It is made up of laminated construction to keep eddy current loss as low as possible.

A single circular lamination used for the construction of the armature core is shown below.

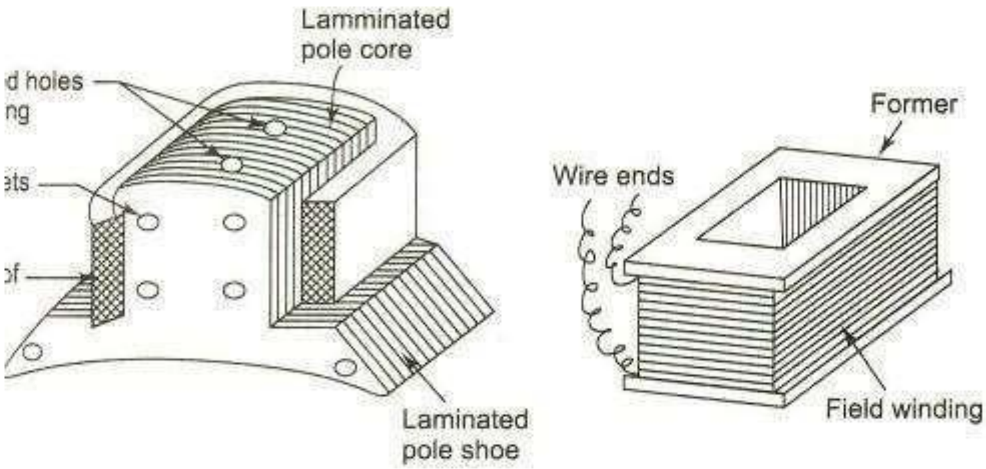
2. Armature winding: Armature winding is nothing but the inter connection of the armature conductors, placed in the slots provided on the armature core. When the armature is rotated, in case of generator magnetic flux gets cut by armature conductors and emf gets induced in them.

Function:

1. Generation of emf takes place in the armature winding in case of generators.
2. To carry the current supplied in case of dc motors.
3. To do the useful work it the external circuit.

Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Field winding: The field winding is wound on the pole core with a definite direction.



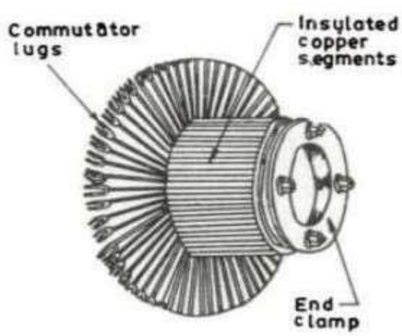

Functions: To carry current due to which pole core on which the winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called 'Field winding' or 'Exciting winding'.

Choice of material : As it has to carry current it should be made up of some conducting material like the aluminum or copper.

But field coils should take any type of shape should bend easily, so copper is the proper choice. Field winding is divided into various coils called as field coils. These are connected in series with each other and wound in such a direction around pole cores such that alternate N and S poles are formed.

Commutator: The rectification in case of dc generator is done by device called as commutator.

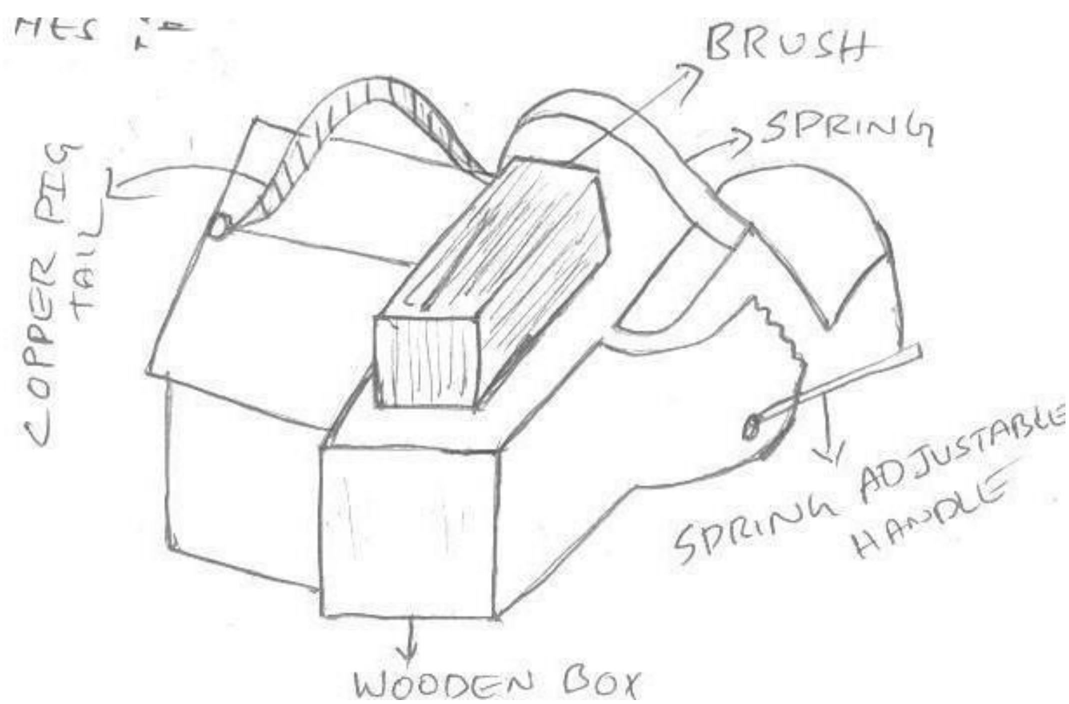



Functions: 1. To facilitate the collection of current from the armature conductors.

2. To convert internally developed alternating emf to in directional (dc) emf
3. To produce unidirectional torque in case of motor.

Choice of material: As it collects current from armature, it is also made up of copper segments. It is cylindrical in shape and is made up of wedge shaped segments which are insulated from each other by thin layer of mica.

Brushes and brush gear: Brushes are stationary and rest on the surface of the Commutator. Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pigtail is used to connect the brush to the external circuit.




Functions: To collect current from commutator and make it available to the stationary external circuit.

Choice of material: Brushes are normally made up of soft material like carbon.

Bearings: Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

Working of DC generator:

The generator is provided with a magnetic field by sending dc current through the field coils mounted on laminated iron poles and through armature winding.

A short air gap separates the surface of the rotating armature from the stationary pole surface. The magnetic flux coming out of one or more worth poles crossing the air gap , passes through the armature near the gap into one or more adjacent south poles.

The direct current leaves the generator at the positive brush, passes through the circuit and returns to the negative brush.

The terminal voltage of a dc generator may be increased by increasing the current in the field coil and may be reduced by decreasing the current.

Generators are generally run at practically constant speed by their prime mores.

## Types of armature winding:

Armature conductors are connected in a specific manner called as armature winding and according to the way of connecting the conductors; armature winding is divided into two types.

Lap winding: In this case, if connection is started from conductor in slot 1 then the connections overlap each other as winding proceeds, till starting point is reached again.

There is overlapping of coils while proceeding. Due to such connection, the total number of conductors get divided into ‘P’ number of parallel paths, where

P = number of poles in the machine.

Large number of parallel paths indicates high current capacity of machine hence lap winding is pertained for high current rating generators.

Wave winding: In this type, winding always travels ahead avoiding over lapping. It travels like a progressive wave hence called wave winding.

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of machine.

As number of parallel paths is less, it is preferable for low current, high voltage capacity generators.

| Sl. No. | Lap winding | Wave winding |
|---|---|---|
| 1. | Number of parallel paths (A) = poles (P) | Number of parallel paths (A) = 2 (always) |
| 2. | Number of brush sets required is equal to number of poles | Number of brush sets required is always equal to two |
| 3. | Preferable for high current, low voltage capacity generators | Preferable for high current, low current capacity generators |
| 4. | Normally used for generators of capacity more than 500 A | Preferred for generator of capacity less than 500 A. |

## EMF equation of a generator

Let P = number of poles

Ø = flux/pole in webers

Z = total number of armature conductors.

= number of slots x number of conductors/slot

N = armature rotation in revolutions (speed for armature) per minute (rpm)

A = No. of parallel paths into which the ‘z’ no. of conductors are divided.

E = emf induced in any parallel path

$E_g$ = emf generated in any one parallel path in the armature.

Average emf generated/conductor = dØ/dt volt

Flux current/conductor in one revolution

dt = d x p

In one revolution, the conductor will cut total flux produced by all poles = d x p

No. of revolutions/second = N/60

Therefore, Time for one revolution, dt = 60/N second

According to Faraday’s laws of Electromagnetic Induction, emf generated/conductor = dØ/dt= x p x N / 60 volts

This is emf induced in one conductor.

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors in (series)in one path = Z/2

EMF generated/path = ØPN/60 x Z/2 = ØZPN/120 volt

For a simple lap-wound generator

Number of parallel paths = P

Number of conductors in one path = Z/P

EMF generated/path = ØPN/60 (Z/P) =

ØZN/60 A = 2 for simplex – wave winding

A =P for simplex lap-winding

# Armature Reaction and Commutation

## Introduction

In a d.c. generator, the purpose of field winding is to produce magnetic field (called main flux) whereas the purpose of armature winding is to carry armature current. Although the armature winding is not provided for the purpose of producing a magnetic field, nevertheless the current in the armature winding will also produce magnetic flux (called armature flux). The armature flux distorts and weakens the main flux posing problems for the proper operation of the d.c. generator. The action of armature flux on the main flux is called armature
reaction.

## Armature Reaction

So far we have assumed that the only flux acting in a d.c. machine is that due to the main poles called main flux. However, current flowing through armature conductors also creates a magnetic flux (called armature flux) that distorts and weakens the flux coming from the poles. This distortion and field weakening takes place in both generators and motors. The action of armature flux on the main flux is known as armature reaction.
The phenomenon of armature reaction in a d.c. generator is shown in Fig.(2.1)

Only one pole is shown for clarity. When the generator is on no-load, a small current flowing in the armature does not appreciably affect the main flux f1 coming from the pole [See Fig 2.1 (i)]. When the generator is loaded, the current flowing through armature conductors sets up flux f1. Fig. (2.1) (ii) shows flux due to armature current alone. By superimposing f1 and f2, we obtain the resulting flux f3 as shown in Fig. (2.1) (iii). Referring to Fig (2.1) (iii), it is clear that flux density at; the trailing pole tip (point B) is increased while at the leading pole tip (point
4. it is decreased. This unequal field distribution produces the following twoeffects:

The main flux is distorted.

Due to higher flux density at pole tip B, saturation sets in. Consequently, the increase in flux at pole tip B is less than the decrease in flux under pole tip A. Flux f3 at full load is, therefore, less than flux f1 at no load. As we shall see, the weakening of flux due to armature reaction depends upon the position of brushes.

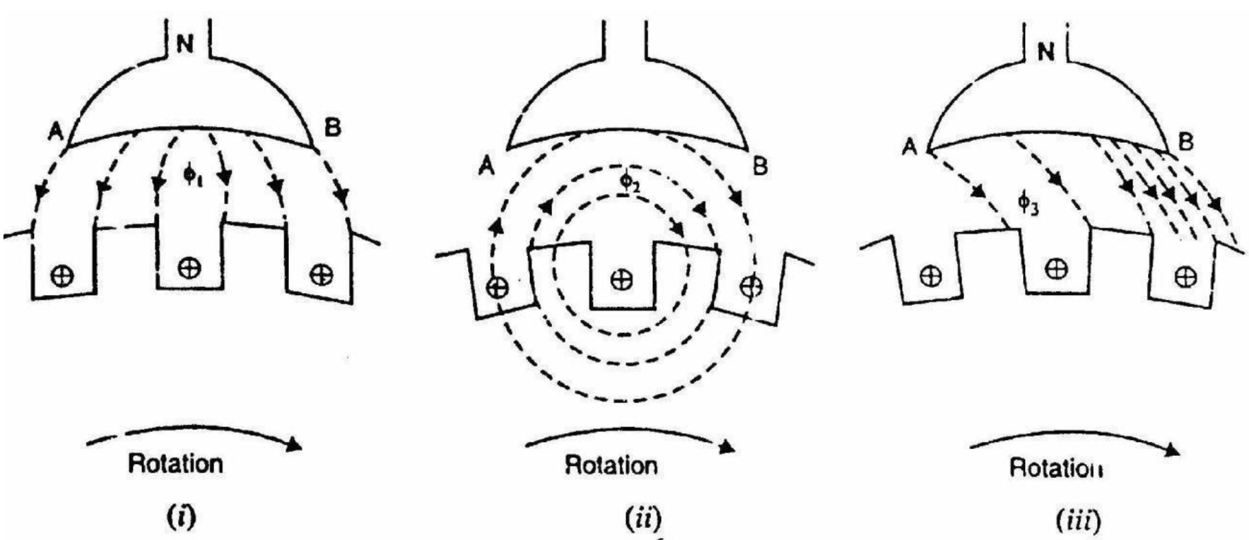



Fig. (2.1)

## Geometrical and Magnetic Neutral Axes

4. The geometrical neutral axis (G.N.A.) is the axis that bisects the angle between the centre line of

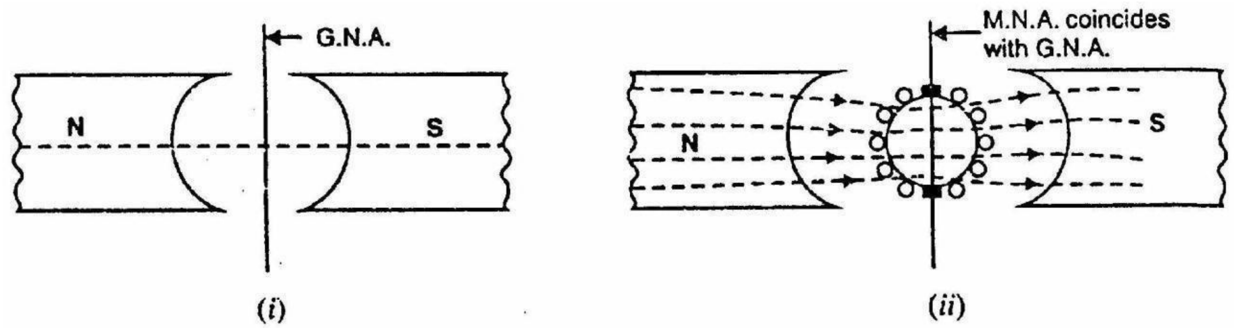



Fig. (2.2)

4. The magnetic neutral axis (M. N. A.) is the axis drawn perpendicular to the mean direction of the flux passing through the centre of the armature. Clearly, no e.m.f. is produced in the armature conductors along this axis because then they cut no flux. With no current in the armature conductors, the M.N.A. coincides with G, N. A. as shown in Fig. (2.2).

5. In order to achieve sparkless commutation, the brushes must lie along M.N.A.

## Explanation of Armature Reaction

With no current in armature conductors, the M.N.A. coincides with G.N.A. However, when current flows in armature conductors, the combined action of main flux and armature flux shifts the M.N.A. from G.N.A. In case of a generator, the M.N.A. is shifted in the direction of rotation of the machine. In order to achieve sparkless commutation, the brushes have to be moved along the new M.N.A. Under such a condition, the armature reaction produces the following two effects:

1. It demagnetizes or weakens the main flux.
2. It cross-magnetizes or distorts the main flux.

Let us discuss these effects of armature reaction by considering a 2-pole generator (though the following remarks also hold good for a multipolar generator).

(i) Fig. (2.3) (i) shows the flux due to main poles (main flux) when the armature conductors carry no current. The flux across the air gap is uniform. The m.m.f. producing the main flux is represented in magnitude and direction by the vector OFm in Fig. (2.3) (i). Note that OFm is perpendicular to G.N.A.

(ii) Fig. (2.3) (ii) shows the flux due to current flowing in armature conductors alone (main poles unexcited). The armature conductors to the left of G.N.A. carry current “in” (′) and those to the right carry current “out” (•). The direction of magnetic lines of force can be found by cork screw rule. It is clear that armature flux is directed downward parallel to the brush axis. The m.m.f. producing the armature flux is represented in magnitude and direction by the vector OFA in Fig. (2.3) (ii).

(iii) Fig. (2.3) (iii) shows the flux due to the main poles and that due to current in armature conductors acting together. The resultant m.m.f. OF is the vector sum of OFm and OFA as shown in Fig. (2.3) (iii). Since M.N.A. is always perpendicular to the resultant m.m.f., the M.N.A. is shifted through an angle q. Note that M.N.A. is shifted in the direction of rotation of the generator.

(iv) In order to achieve sparkless commutation, the brushes must lie along the M.N.A. Consequently, the brushes are shifted through an angle q so as to lie along the new M.N.A. as shown in Fig. (2.3) (iv). Due to brush shift, the m.m.f. FA of the armature is also rotated through the same angle q. It is because some of the conductors which were earlier under N-pole now come under S-pole and vice-versa. The result is that armature m.m.f. FA will no longer be vertically downward but will be rotated in the direction of rotation through an angle q as shown in Fig. (2.3) (iv). Now FA can be resolved into

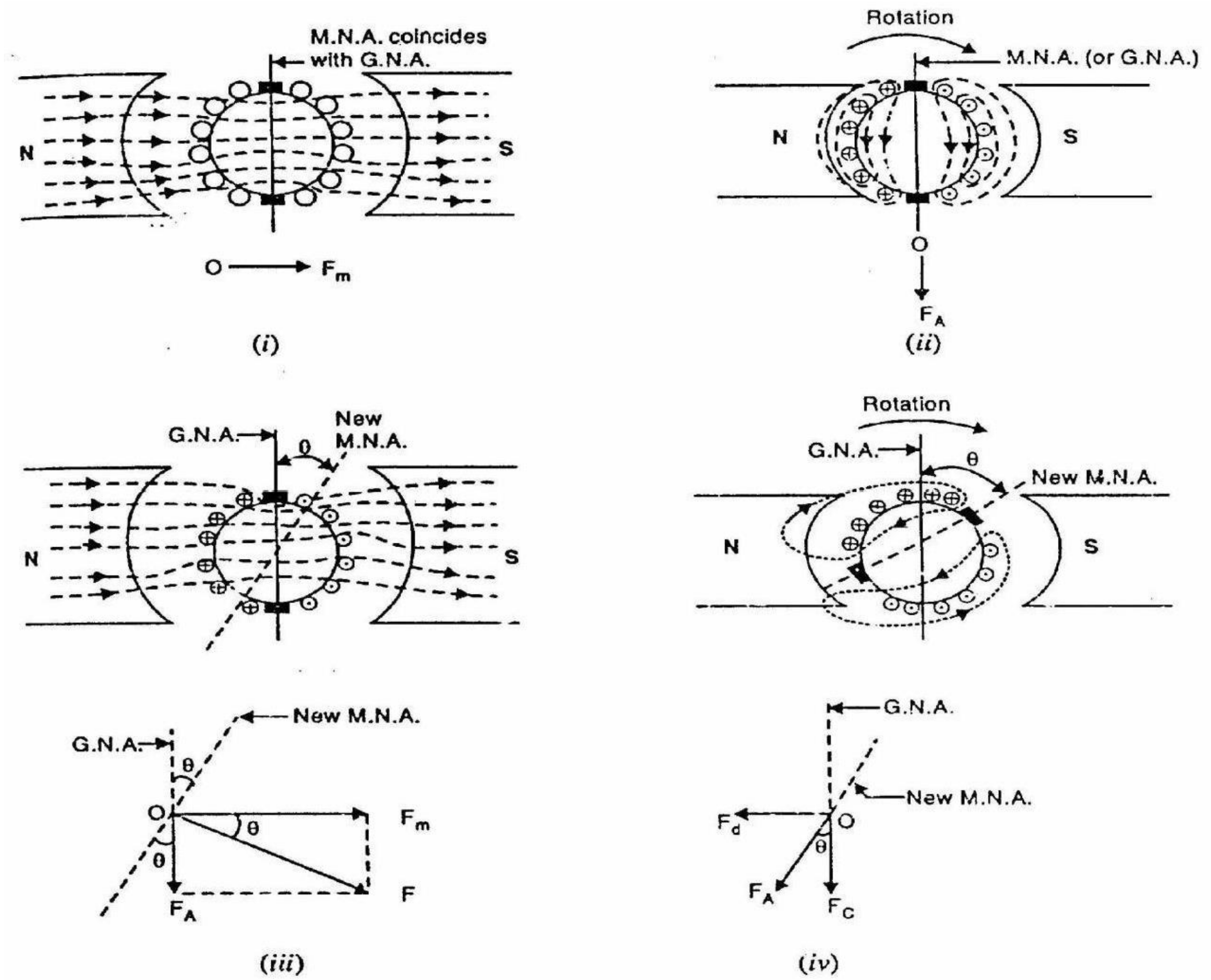



Fig. (2.3)

(a) The component Fd is in direct opposition to the m.m.f. OFm due to main poles. It has a demagnetizing effect on the flux due to main poles. For this reason, it is called the demagnetizing or weakening component of armature reaction.

(b) The component Fc is at right angles to the m.m.f. OFm due to main poles. It distorts the main field. For this reason, it is called the cross magnetizing or distorting component of armature reaction. It

may be noted that with the increase of armature current, both demagnetizing and distorting effects will increase.

Conclusions

(i) With brushes located along G.N.A. (i.e., q = 0°), there is no demagnetizing component of armature reaction (Fd = 0). There is only distorting or cross magnetizing effect of armature reaction.

(ii) With the brushes shifted from G.N.A., armature reaction will have both demagnetizing and distorting effects. Their relative magnitudes depend on the amount of shift. This shift is directly proportional to the Armature current.

(iii) The demagnetizing component of armature reaction weakens the main flux. On the other hand, the distorting component of armature reaction distorts the main flux.

(iv) The demagnetizing effect leads to reduced generated voltage while cross magnetizing effect leads to sparking at the brushes.

## Demagnetizing and Cross-Magnetizing Conductors

With the brushes in the G.N.A. position, there is only cross-magnetizing effect of armature reaction. However, when the brushes are shifted from the G.N.A. position, the armature reaction will have both demagnetizing and cross magnetizing effects. Consider a 2-pole generator with brushes shifted (lead) $\theta m$ mechanical degrees from G.N.A. We shall identify the armature conductors that produce demagnetizing effect and those that produce cross-magnetizing effect.

(i) The armature conductors $\theta m$ on either side of G.N.A. produce flux in direct opposition to main flux as shown in Fig. (2.4) (i). Thus the conductors lying within angles AOC = BOD = 2 $\theta m$ at the top and bottom of the armature produce demagnetizing effect. These are called demagnetizing armature conductors and constitute the demagnetizing ampere-turns of armature reaction (Remember two conductors constitute a turn).

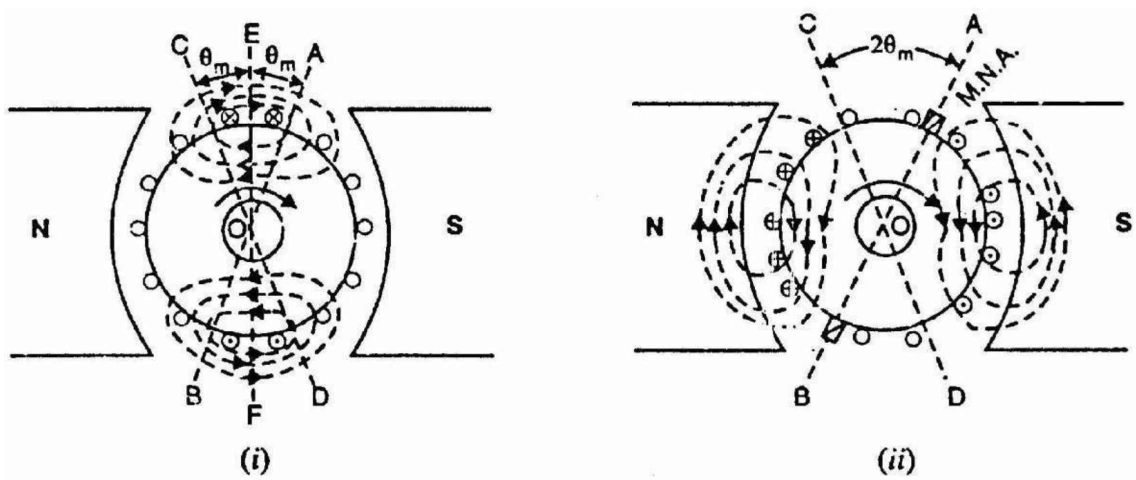



Fig.(2.4)

(ii) The axis of magnetization of the remaining armature conductors lying between angles AOD and COB is at right angles to the main flux as shown in Fig. (2.4) (ii). These conductors produce the cross-magnetizing (or distorting) effect i.e., they produce uneven flux distribution on each pole. Therefore, they are called cross-magnetizing conductors and constitute the cross-magnetizing ampere-turns of armature reaction.

## Calculation of Demagnetizing Ampere-Turns Per Pole (ATd/Pole)

It is sometimes desirable to neutralize the demagnetizing ampere-turns of armature reaction. This is achieved by adding extra ampere-turns to the main field winding. We shall now calculate the demagnetizing ampere-turns per pole (ATd/pole).

Let Z = total number of armature conductors

I = current in each armature Conductor

= Ia/2 ... for simplex wave winding

= Ia/P ... for simplex lap winding

$\theta m$ = forward lead in mechanical degrees

Referring to Fig. (2.4) (i) above, we have,
Total demagnetizing armature conductors

$$= \text{Conductors in angles AOC and BOD} = \frac{4\theta_m}{360} \times Z$$

Since two conductors constitute one turn,

$$\therefore \quad \text{Total demagnetizing ampere-turns } = \frac{1}{2}\left[\frac{4\theta_m}{360} \times Z\right] \times I = \frac{2\theta_m}{360} \times ZI$$

These demagnetizing ampere-turns are due to a pair of poles.

$$\therefore \quad \text{Demagnetizing ampere-turns/pole} = \frac{\theta_m}{360} \times ZI$$

i.e., $$AT_d\,/\,\text{pole} = \frac{\theta_m}{360} \times ZI$$

As mentioned above, the demagnetizing ampere-turns of armature reaction can be neutralized by putting extra turns on each pole of the generator.

$$\therefore \quad \text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \qquad \text{for a shunt generator}$$

$$= \frac{AT_d}{I_a} \qquad \text{for a series generator}$$

Note. When a conductor passes a pair of poles, one cycle of voltage is generated. We say one cycle contains 360 electrical degrees. Suppose there are P poles in a generator. In one revolution, there are 360 mechanical degrees and 360 *P/2 electrical degrees.

$$\therefore \quad 360^\circ \text{ mechanical} = 360 \times \frac{P}{2} \text{ electrical degrees}$$

or $$1^\circ \text{ Mechanical} = \frac{P}{2} \text{ electrical degrees}$$

$$\therefore \quad \theta \text{ (mechanical)} = \frac{\theta\text{(electrical)}}{\text{Pair of pols}}$$

or $$\theta_m = \frac{\theta_e}{P/2} \qquad \therefore \quad \theta_m = \frac{2\theta_e}{P}$$

Cross-Magnetizing Ampere-Turns Per Pole (ATc/Pole)

We now calculate the cross-magnetizing ampere-turns per pole (ATc/pole).

Total armature reaction ampere-turns per pole

$$= \frac{Z/2}{P} \times I = \frac{Z}{2P} \times I \qquad (\because \text{ two conductors make one turn})$$

Demagnetizing ampere-turns per pole is given by;

$$AT_d\,/\,\text{pole} = \frac{\theta_m}{360} \times ZI$$

(found as above)

Cross-magnetizing ampere-turns/pole are

$$AT_d\,/\,\text{pole} = \frac{Z}{2P} \times I - \frac{\theta_m}{360} \times ZI = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$\therefore \quad AT_d\,/\,\text{pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

## Compensating Windings

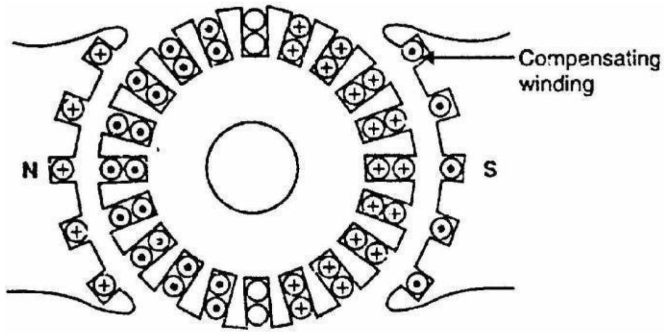



Fig. (2.5)

The cross-magnetizing effect of armature reaction may cause trouble in d.c. machines subjected to large fluctuations in load. In order to neutralize the cross magnetizing effect of armature reaction, a compensating winding is used. A compensating winding is an auxiliary winding embedded in slots in the pole faces as shown in Fig. (2.5). It is connected in series with armature in a manner so that the direction of current through the compensating conductors in any one pole face will be opposite to the direction of the current through the adjacent armature conductors [See Fig. 2.5].

Let us now calculate the number of compensating conductors/ pole face. In calculating the conductors per pole face required for the compensating winding, it should be remembered that the current in the compensating conductors is the armature current Ia whereas the current in armature conductors is Ia/A where A is the number of parallel paths.

| | | | |
|---|---|---|---|
| Let | Zc | = | No. of compensating conductors/pole face |
| | Za | = | No. of active armature conductors |
| | Ia | = | Total armature current |
| | Ia/A | = | Current in each armature conductor |

$$\therefore \quad Z_c I_a = Z_a \times \frac{I_a}{A}$$

or $$Z_c = \frac{Z_a}{A}$$

The use of a compensating winding considerably increases the cost of a machine and is justified only for machines intended for severe service e.g., for high speed and high voltage machines.

## AT/Pole for Compensating Winding

Only the cross-magnetizing ampere-turns produced by conductors under the pole face are effective in producing the distortion in the pole cores. If Z is the total number of armature conductors and P is the number of poles, then,

$$\text{No. of armature conductors/pole} = \frac{Z}{P}$$

$$\text{No. of armature turns/pole} = \frac{Z}{2P}$$

$$\text{No. of armature turns under pole face} = \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

If I is the current through each armature conductor, then,

$$\text{AT/pole required for compensating winding} = \frac{ZI}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

$$= \text{Armature AT/pole} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

## Commutation

Fig. (2.6) shows the schematic diagram of 2-pole lap-wound generator. There are two parallel paths between the brushes. Therefore, each coil of the winding carries one half (Ia/2 in this case) of the total current (Ia) entering or leaving the armature.

Note that the currents in the coils connected to a brush are either all towards the brush (positive brush) or all directed away from the brush (negative brush). Therefore, current in a coil will reverse as the coil passes a brush. This reversal of current as the coil passes & brush is called commutation.

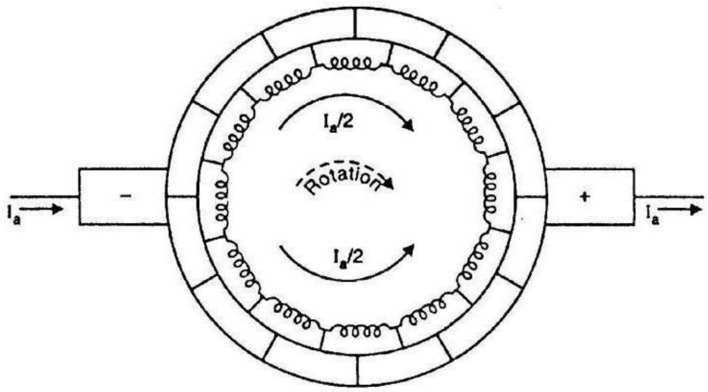



Fig. (2.6)

The reversal of current in a coil as the coil passes the brush axis is called commutation. When commutation takes place, the coil undergoing commutation is short circuited by the brush. The brief period during which the coil remains short circuited is known as commutation period Tc. If the current reversal is completed by the end of commutation period, it is called ideal commutation. If the current reversal is not completed by that time, then sparking occurs between the brush and the commutator which results in progressive damage to both.

## Ideal commutation

Let us discuss the phenomenon of ideal commutation (i.e., coil has no inductance) in one coil in the armature winding shown in Fig. (2.6) above. For this purpose, we consider the coil A. The brush width is equal to the width of one commutator segment and one mica insulation. Suppose the total armature current is 40 A. Since there are two parallel paths, each coil carries a current of 20 A.

(i) In Fig. (2.7) (i), the brush is in contact with segment 1 of the commutator. The commutator segment 1 conducts a current of 40 A to the brush; 20 A from coil A and 20 A from the adjacent coil as shown. The coil A has yet to undergo commutation.

(ii) As the armature rotates, the brush will make contact with segment 2 and thus short-circuits the coil A as shown in Fig. (2.7) (ii). There are now two parallel paths into the brush as long as the short-circuit of coil A exists. Fig. (2.7) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. For this condition, the resistance of the path through segment 2 is three times the resistance of the path through segment 1 (Q contact resistance varies inversely as the area of contact of brush with the segment). The brush again conducts a current of 40 A; 30 A through segment 1 and 10 A through segment 2. Note that current in coil A (the coil undergoing commutation) is reduced from 20 A to 10 A.

(iii) Fig. (2.7) (iii) shows the instant when the brush is one-half on segment 2 and one-half on segment 1. The brush again conducts 40 A; 20 A through segment 1 and 20 A through segment 2 (Q now the resistances of the two parallel paths are equal). Note that now. current in coil A is zero.

(iv) Fig. (2.7) (iv) shows the instant when the brush is three-fourth on segment 2 and one-fourth on segment 1. The brush conducts a current of 40 A; 30 A through segment 2 and 10 A through segment 1. Note that current in coil A is 10 A but in the reverse direction to that before the start of commutation. The reader may see the action of the commutator in

(v) Fig. (2.7) (v) shows the instant when the brush is in contact only with segment 2. The brush again conducts 40 A; 20 A from coil A and 20 A from the adjacent coil to coil A. Note that now current in coil A is 20 A but in the reverse direction. Thus the coil A has undergone commutation. Each coil undergoes commutation in this way as it passes the brush axis. Note that during commutation, the coil under consideration remains short circuited by the brush.

Fig. (2.8) shows the current-time graph for the coil A undergoing commutation. The horizontal line AB represents a constant current of 20 A upto the beginning of commutation. From the finish of commutation, it is represented by another horizontal line CD on the opposite side of the zero line and the same distance from it as AB i.e., the current has exactly reversed (- 20 A). The way in which current changes from B to C depends upon the conditions under which the coil undergoes commutation. If the current changes at a uniform rate (i.e., BC is a straight line), then it is called ideal commutation as shown in Fig. (2.8). Under such conditions, no sparking will take place between the brush and the commutator.

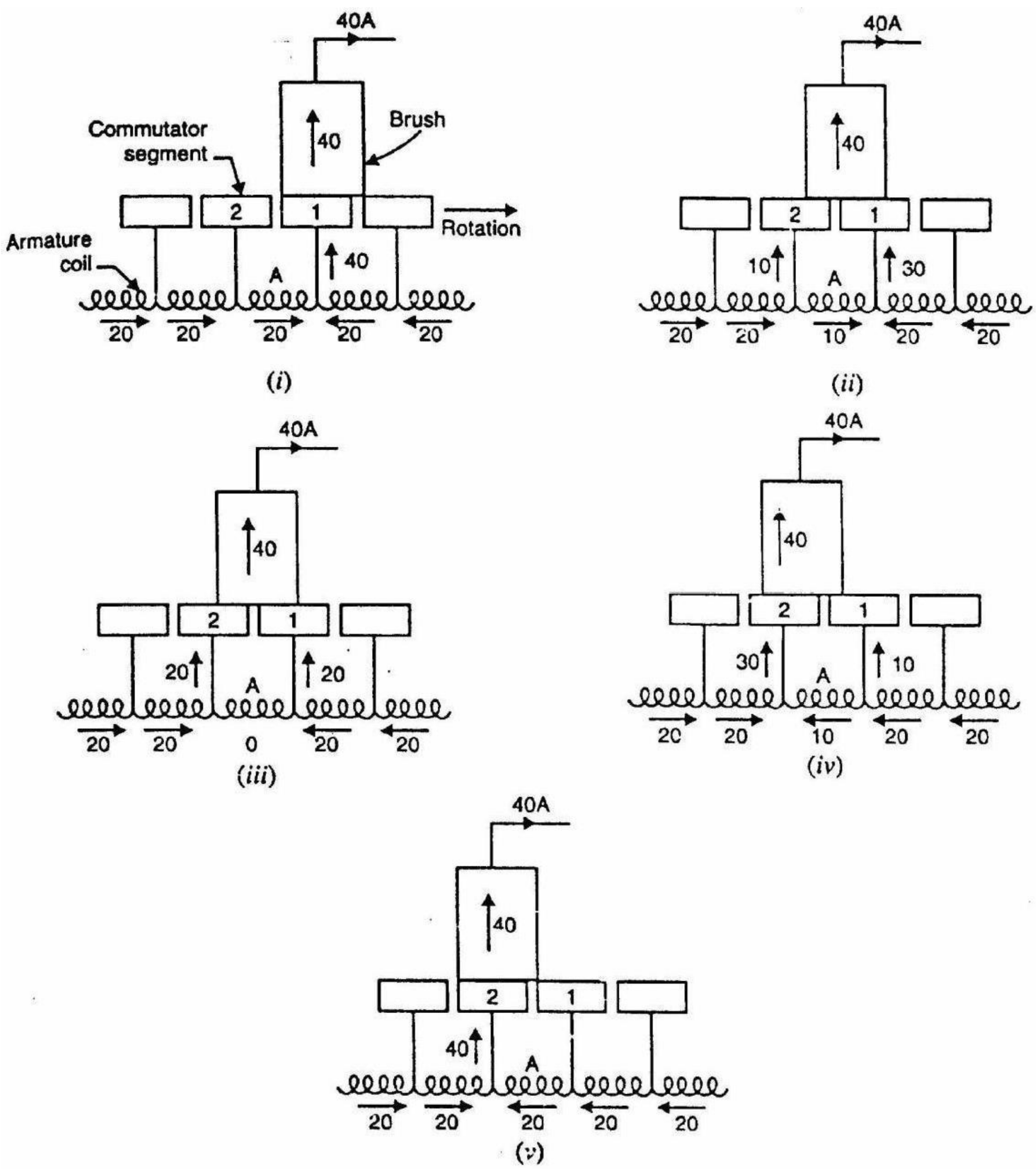



Fig. (2.7)

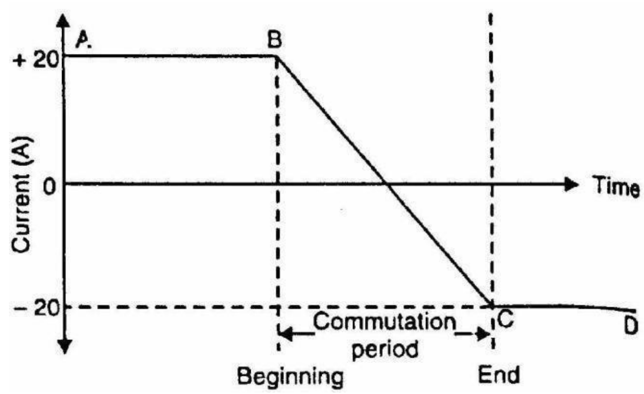



Fig. (2.8)

Practical difficulties

The ideal commutation (i.e., straight line change of current) cannot be attained in practice. This is mainly due to the fact that the armature coils have appreciable inductance. When the current in the coil undergoing commutation changes, self-induced e.m.f. is produced in the coil. This is generally called reactance voltage. This reactance voltage opposes the change of current in the coil undergoing commutation. The result is that the change of current in the coil undergoing commutation occurs more slowly than it would be under ideal commutation.

This is illustrated in Fig. (2.9). The straight line RC represents the ideal commutation whereas the curve BE represents the change in current when self-inductance of the coil is taken into account. Note that current CE (= 8A in Fig. 2.9) is flowing from the commutator segment 1 to the brush at the instant when they part company. This results in sparking just as when any other current carrying circuit is broken. The sparking results in overheating of commutators brush contact and causing damage to both.

Fig. (2.10) illustrates how sparking takes place between the commutators segment and the brush. At the end of commutation or short-circuit period, the current in coil A is reversed to a value of 12 A (instead of 20 A) due to inductance of the coil. When the brush breaks contact with segment 1, the remaining 8 A current jumps from segment 1 to the brush through air causing sparking between segment 1 and the brush.

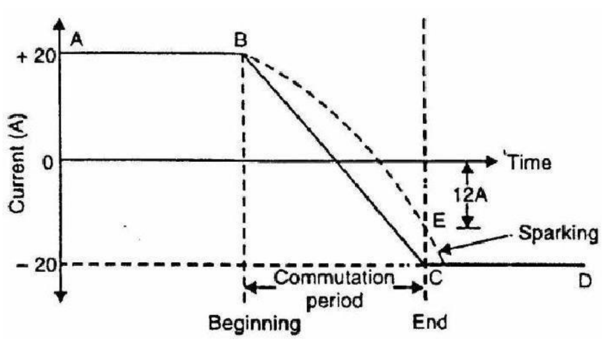



Fig. (2.9)

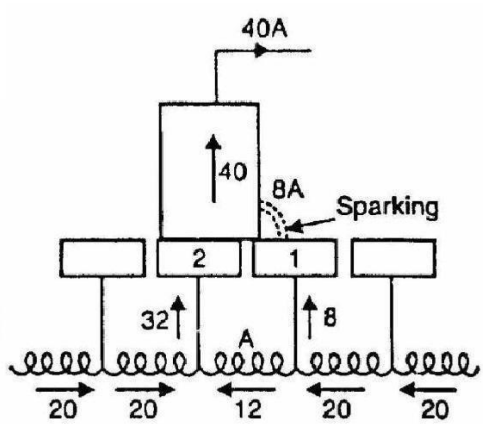



Fig. (2.10)

Calculation of Reactance Voltage

Reactance voltage = Coefficient of self-inductance *Rate of change of current

When a coil undergoes commutation, two commutator segments remain short circuited by the brush. Therefore, the time of short circuit (or commutation period Tc) is equal to the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating strip of mica

Let Wb = brush width in cm;
Wm = mica thickness in cm
v = peripheral speed of commutator in cm/s

$\therefore$ Commutation period, $T_c = \frac{W_b - W_m}{v}$ seconds

The commutation period is very small, say of the order of 1/500 second.

Let the current in the coil undergoing commutation change from + I to – I (amperes) during the commutation. If L is the inductance of the coil, then reactance voltage is given by;

Reactance voltage, $E_R = L*2I/T_c$

## Methods of Improving Commutation

Improving commutation means to make current reversal in the short-circuited coil as sparkless as possible. The following are the two principal methods of improving commutation:

(i) Resistance commutation

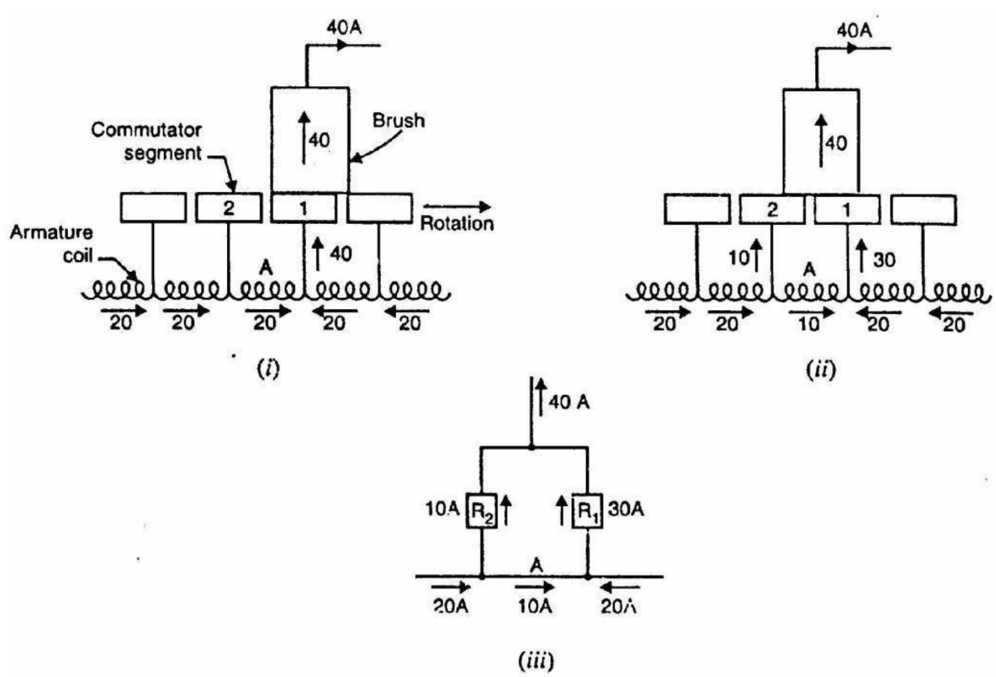
(ii) E.M.F. commutation

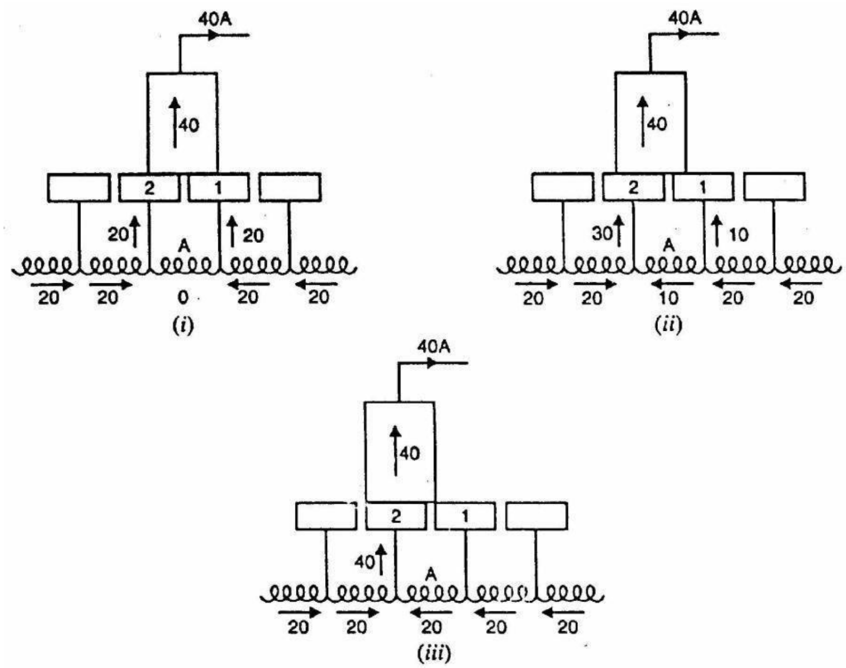
## Resistance Commutation

The reversal of current in a coil (i.e., commutation) takes place while the coil is short-circuited by the brush. Therefore, there are two parallel paths for the current as long as the short circuit exists. If the contact resistance between the brush and the commutator is made large, then current would divide in the inverse ratio of contact resistances (as for any two resistances in parallel). This is the key point in improving commutation. This is achieved by using carbon brushes (instead of Cu brushes) which have high contact resistance. This method of improving commutation is called resistance commutation. Figs. (2.11) and (2.12) illustrates how high contact resistance of carbon brush improves commutation (i.e., reversal of current) in coil A.

In Fig. (2.11) (i), the brush is entirely on segment 1 and, therefore, the current in coil A is 20 A. The coil A is yet to undergo commutation. As the armature rotates, the brush short circuits the coil A and there are two parallel paths for the current into the brush.

Fig. (2.11) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. The equivalent electric circuit is shown in Fig. (2.11) (iii) where R1 and R2 represent the brush contact resistances on segments 1 and 2. A resistor is not shown for coil A since it is assumed that the coil resistance is negligible as compared to the brush contact resistance





40A
40
2
1
20
A
20
20
20
0
20
20
(i)
40A
40
2
1
30
A
10
20
20
10
20
20
(ii)
40A
40
2
1
40
A
20
20
20
20
20
(iii)

The values of current in the parallel paths of the equivalent circuit are determined by the respective resistances of the paths. For the condition shown in Fig. (2.11) (ii), resistor R2 has three times the resistance of resistor R1. Therefore, the current distribution in the paths will be as shown. Note that current in coil A is reduced from 20 A to10 A due to division of current in (he inverse ratio of contact resistances. If the Cu brush is used (which has low contact resistance), R1 R2 and the current in coil A would not have reduced to 10 A.

As the carbon brush passes over the commutator, the contact area with segment 2 increases and that with segment 1 decreases i.e., R2 decreases and R1 increases. Therefore, more and more current passes to the brush through segment 2. This is illustrated in Figs. (2.12) (i) and (2.12) (ii), When the break between the brush and the segment 1 finally occurs [See Fig. 2.12 (iii)], the current in the coil is reversed and commutation is achieved. It may be noted that the main cause of sparking during commutation is the production of reactance voltage and carbon brushes cannot prevent it.

Nevertheless, the carbon brushes do help in improving commutation. The other minor advantages of carbon brushes are:

(i) The carbon lubricates and polishes the commutator.

(ii) If sparking occurs, it damages the commutator less than with copper brushes and the damage to the brush itself is of little importance.

E.M.F. Commutation

In this method, an arrangement is made to neutralize the reactance voltage by producing a reversing voltage in the coil undergoing commutation. The reversing voltage acts in opposition to the reactance voltage and neutralizes it to some extent. If the reversing voltage is equal to the reactance voltage, the effect of the latter is completely wiped out and we get sparkless commutation. The reversing voltage may be produced in the following two ways:

(i) By brush shifting

(ii) By using interpoles or compoles

(i) By brush shifting

In this method, the brushes are given sufficient forward lead (for a generator) to bring the short-circuited coil (i.e., coil undergoing commutation) under the influence of the next pole of opposite polarity. Since the short-circuited coil is now in the reversing field, the reversing voltage produced cancels the reactance voltage. This method suffers from the following drawbacks:

(a) The reactance voltage depends upon armature current. Therefore, the brush shift will depend on the magnitude of armature current which keeps on changing. This necessitates frequent shifting of brushes.

(b) The greater the armature current, the greater must be the forward lead for a generator. This increases the demagnetizing effect of armature reaction and further weakens the mainfield.

(ii) By using interpoles or compotes

The best method of neutralizing reactance voltage is by, using interpoles or compoles.

## Interpoles or Compoles

The best way to produce reversing voltage to neutralize the reactance voltage is by using interpoles or compoles. These are small poles fixed to the yoke and spaced mid-way between the main poles (See Fig. 2.13). They are wound with comparatively few turns and connected in series with the armature so that they carry armature current. Their polarity is the same as the next main pole ahead in the direction of rotation for a generator (See Fig. 2.13). Connections for a d.c. generator with interpoles is shown in Fig. (2.14).

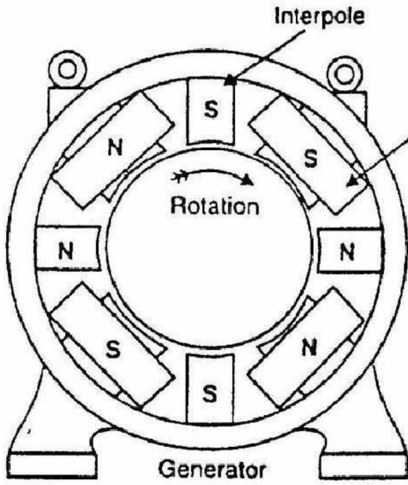



Fig. (2.13)

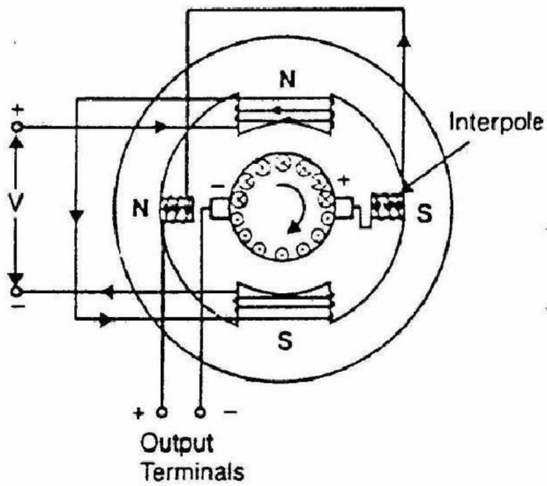



Fig. (2.14)

## Functions of Interpoles

The machines fitted with interpoles have their brushes set on geometrical neutral axis (no lead). The interpoles perform the following two functions:

(i) As their polarity is the same as the main pole ahead (for a generator), they induce an e.m.f. in the coil (undergoing commutation) which opposes reactance voltage. This leads to sparkless commutation. The e.m.f. induced by compoles is known as commutating or reversing e.m.f. Since the interpoles carry the armature current and the reactance voltage is also proportional to armature current, the neutralization of reactance voltage is automatic.

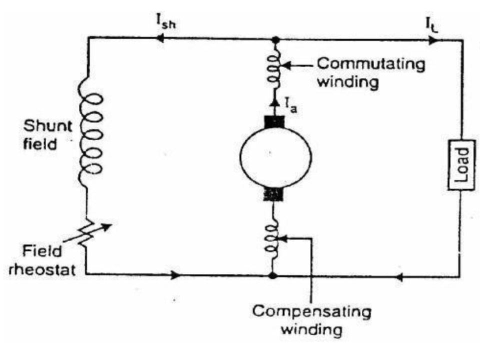



Fig. (2.15)

(ii) The m.m.f. of the compoles neutralizes the cross-magnetizing effect of armature reaction in small region in the space between the main poles. It is because the two m.m.f.s oppose each other in this region. Fig. (2.15) shows the circuit diagram of a shunt generator with commutating winding and compensating winding. Both these windings are connected in series with the armature and so they carry the armature current. However, the functions they perform must be understood clearly. The main function of commutating winding is to produce reversing (or commutating) e.m.f. in order to cancel the reactance voltage. In addition to this, the m.m.f. of the commutating winding neutralizes the cross magnetizing ampere-turns in the space between the main poles. The compensating winding neutralizes the cross- magnetizing effect of armature reaction under the pole faces.

# UNIT – III

# Types of D.C Generators & Load Characteristics

- Methods of Excitation
- Separately excited and self excited generators
- Build-up of E.M.F, Critical field resistance and Critical speed
- Causes for failure to self excite and remedial measures
- Load characteristics of shunt, series and compound generators
- Parallel operation of DC series generators
- Use of equalizer bar and cross connection of field windings
- Load sharing
    - **Important concepts and Formulae:**
    - Illustrative examples

**EMF Equation:**

This being very important for understanding of a Generator performance we will derive a detailed expression for the exact induced emf in a generator in terms of all the following DC Machine parameters**.**

**Ø** The flux from a pole (webers)

**Z** The total number of conductors on the armature

**a** The number of parallel paths

- In a practical machine all the conductors are not connected in series. They are divided into groups of parallel conductors and then all the groups are connected in series to get higher voltage. In each group there are **'a'** conductors in parallel and hence there are **'a'** parallel current paths and each parallel path will have **Z/a** conductors in series.

**N** The Speed of rotation (RPM)

**ω** The speed (Radians/sec)

**P** The number of poles

Now consider one conductor on the armature. As this conductor makes one complete revolution it cuts **PØ** webers of flux.

Since the induced emf in a conductor is its rate of cutting of flux lines ( Rate of change of Flux linkage ) the emf **'e'** induced in such a single conductor is equal to

**e = PØ/ Time for one revolution in seconds = PØ/(60/N) = NPØ/60 volts**

There are **Z/a** conductors in series in each parallel path.

∴ the total induced emf **'E' = (Z/a) NPØ/60 = ( NPØ Z )/ (a. 60)**

$$E_A = (Ø\ ZN/60).(P/a)$$

The armature conductors are generally connected in two methods. Viz. Lap winding and Wave winding**.**

In Lap wound machines the number of parallel paths **'a' = P** ∴ **'E' = (Ø ZN/60)**

In Wave wound machines the number of parallel paths **'a' = 2** ∴ **'E' = (Ø ZN/60).( P/2)**

In general the emf induced in a DC machine can be represented as $E_A = K_a.\ Ø.N$

Where $K_a = ZP/60.a$

Sometimes it is convenient to express the emf induced in terms of the angular rotation **ω (Rad/sec)** and then the expression for emf becomes:

$E_A = (Ø\ ZN/60).(P/a) = (ZP/a).\ Ø.\ N/60 = (ZP/.a).\ Ø.\ (ω/2π) = (ZP/2πa).Ø.ω = K_a.\ Ø.ω$

**(since N/60 RPS = 2π. N/60 Radians /sec = ω Radians /sec and ∴ N/60 = ω/2π)**

Where $K_a$ is the generalized constant for the DC machine's armature and is given by :

$$K_a = (ZP/2πa)$$

Where **Ø** is the flux/per pole in the machine (Webers), **N** is the speed of rotation (RPM) **ω** is the angular speed (Radians/sec) and $K_a$ is a constant depending on the machine parameters.

*And thus finally* $E_A = K_a.\ Ø.\ ω$ *and we can say in general, the induced voltage in any DC machine will depend on the following three factors:*

*1. The flux* **Ø** *in the machine*

*2. The angular speed of rotation* **ω** *and*

*3. A constant representing the construction of the machine.* ***(ZP/2πa)***

*(i.e. the number of conductors* ***'Z'****, the number of poles* ***'P'*** *and the number of parallel paths* ***'a'*** *along with the other constant* ***'2π'****)*

**Important Aspects of DC Generators:**

- The terminal characteristic of a DC Machine is a plot of the output quantities of the Machine against each other. For a DC Generator the output quantities are the Terminal Voltage and the Line (Load) current.
- The various types of Generators differ in their terminal characteristics (Voltage-Current ) and therefore to the application to which they are suited.
- The DC Generators are compared by their Voltages, Power ratings, their efficiencies and Voltage regulation. Voltage Regulation **(VR)** is defined by the equation: $V_R = [(V_{nl} - V_{fl}) / V_{fl}].100\ \%$ Where $V_{nl}$ is the No load terminal voltage and $V_{fl}$ is the Full load terminal voltage. It is a rough measure of the Generator's Voltage- Current Characteristic. A positive voltage regulation means a drooping characteristic and a negative regulation means a rising characteristic.
- Since the speed of the prime movers affects the Generator voltage and prime movers can have varying speed characteristics, The voltage regulation and speed characteristics

of the Generator are always compared assuming that the ***Prime mover's speed is always constant***.

## Methods of excitation: (The method by which the field current is generated)

The performance characteristics of a dc machine are greatly influenced by the way in which the field winding is excited with direct current. There are two basic ways of exciting a dc machine.

**1. Separate excitation:** The field is excited from a separate and independent DC source as shown in fig(a) below. It is flexible as full and independent control of both Field and Armature circuits is possible.

**2. Self- excitation:** The field is excited either from its own armature voltage (Shunt Excitation: fig-b) or own armature current (Series excitation : fig-c)

The dc machine excitation is also classified in three other ways:

**1. Shunt excitation :** Here the field winding is excited in parallel with armature circuit and hence the name shunt excitation. It is provided with a large number (hundreds or even thousands) of turns of thin wire and therefore, has a high resistance and carries a small current. Since the armature voltage of a dc machine remains substantially constant, the shunt field could be regulated by placing an external series resistance in its circuit.

**2. Series excitation :** Here the field winding has a few turns of thick wire and is excited with armature current by placing it in series with armature, and therefore it is known as series field winding. For a given field current, control of this field is achieved by means of a diverter, a low resistance connected in parallel to series winding. A more practical way of a series field control is changing the number of turns of the winding by suitable tappings which are brought out for control purpose.

**3. Compound Excitation:** In compound excitation both shunt and series fields are excited. If the two fields aid each other such that the resultant air gap flux per pole is increased (their ampere turns are additive), then the excitation is called ***cumulative compound excitation*** as shown in Fig. (d). If the series field flux opposes the shunt field flux such that the resultant air gap flux per pole is decreased, then the excitation is called ***differential compound excitation*** as shown in Fig. (e). The series field is so designed that the increase or decrease in flux/pole is to a limited extent.

Further there are two types of compounding connections. ***Long Shunt*** and ***Short shunt*** . In long shunt compound of Fig. (f ) the shunt field is connected across the output terminals. In short shunt compound, the shunt field is connected directly across the armature as shown in Fig. (g). There is no significant difference in machine performance for the two types of connections. The choice between them depends upon mechanical consideration or the reversing switches.

Figure below shows the physical arrangement of shunt and series field windings on one pole of a machine.

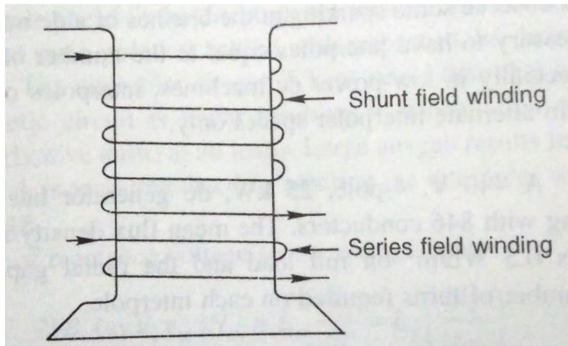
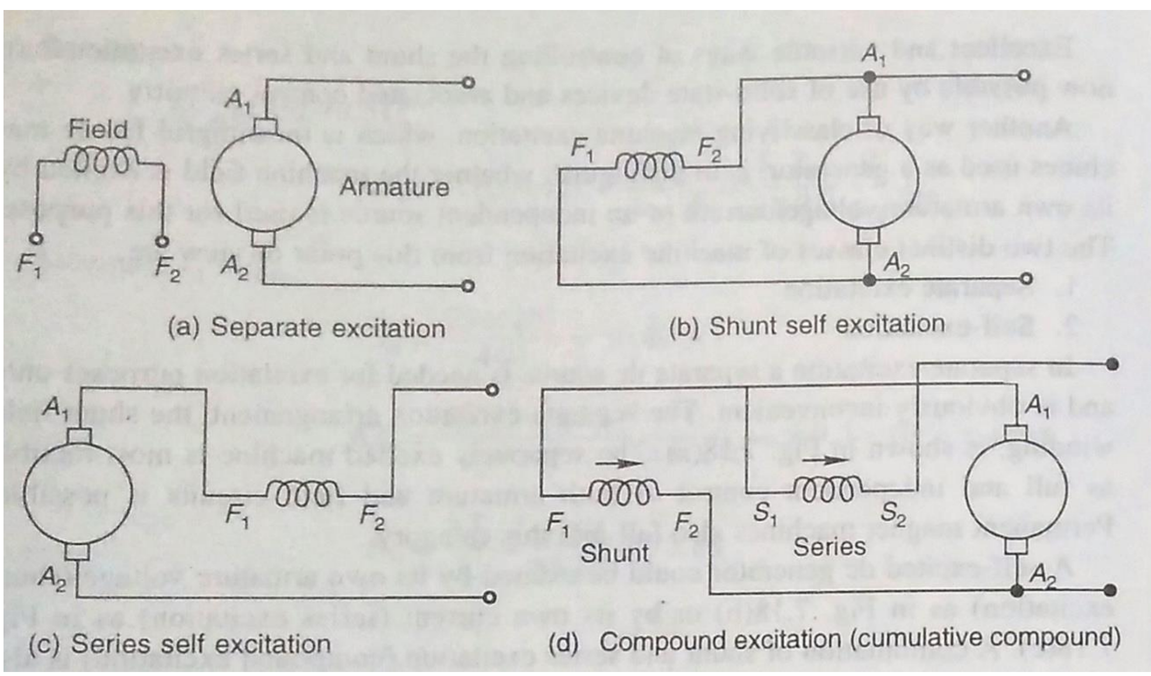
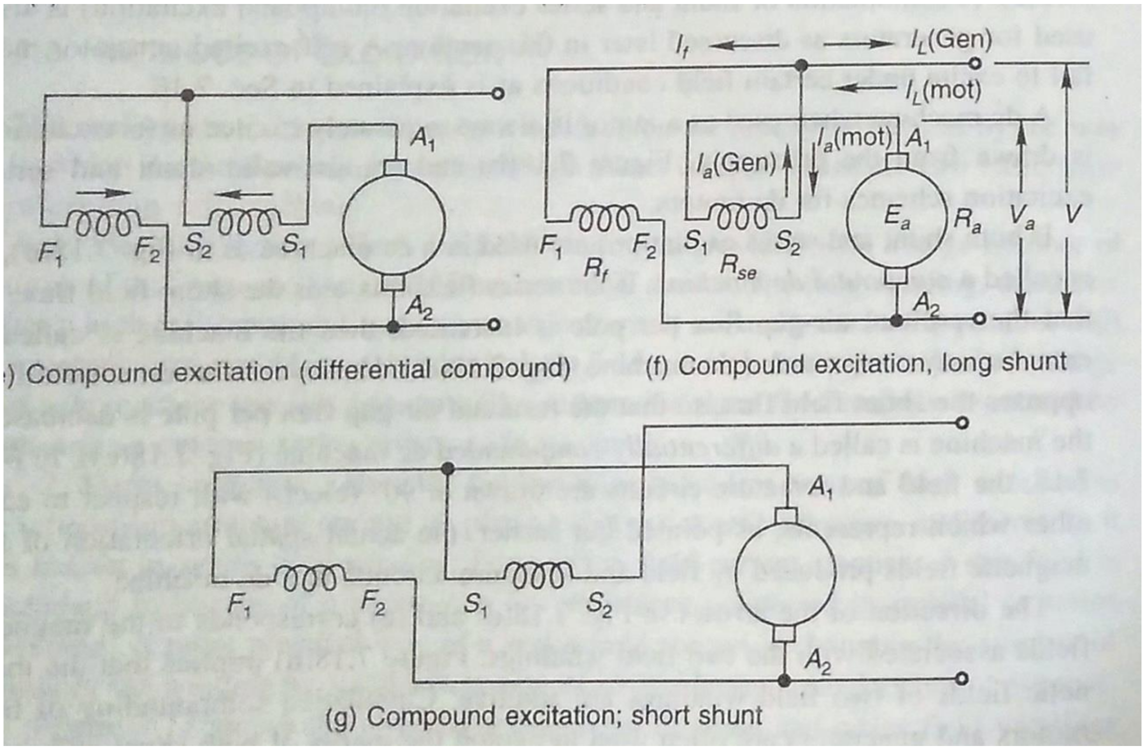



Fig: Arrangement of shunt and series field windings on one pole of a machine.

Excellent and versatile ways of controlling the shunt and series excitations are now possible by use of solid-state devices and associated control circuitry.

In showing the excitation diagrams of a dc machine, the field winding is shown to be at 90° (electrical) with respect to the armature circuit which is the actual spatial orientation of the magnetic fields produced by the field and armature circuits in a DC machine.






**Magnetization characteristics of DC Generators:**

No load or Open circuit magnetization characteristic of any DC Machine is a plot of the Field flux versus the magnetizing current. Since measurement of field flux is difficult we use the relation for the emf induced in a DC machine **$E_A$ = K. Ø.N** from which we can see that the induced voltage is proportional to the Flux in the machine when the speed is maintained constant. Hence we conduct a test on the given DC machine to obtain data on the induced voltage as a function of the field current.

The diagram of the test setup required to obtain the above data is shown in the figure below.

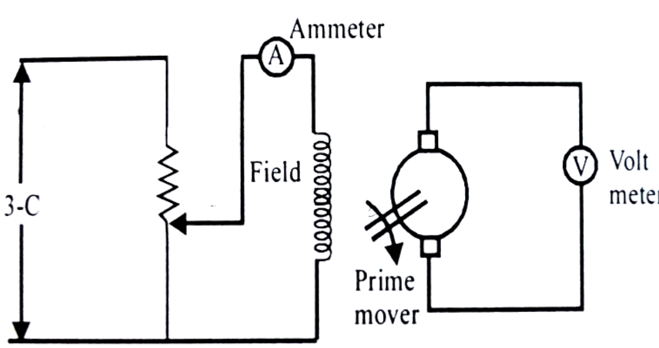



**Fig: Test setup with a DC machine to obtain the No load magnetization Characteristic**

The prime mover gives the required mechanical energy to the DC Machine and it can be a small Diesel engine. The rheostat connected between the DC Input and the field winding is used to adjust and get the required field current. The field current is initially set to Zero and the Armature volatage is measured. Then the field current is gradually increased and the corresponding values of Armature voltage are measured until the output voltage saturates. Next the field current is brought back to zero gradually and the corresponding Armature voltages are measured at a few points. The corresponding data on Armature voltage is plotted against field current and is shown in the figure below.

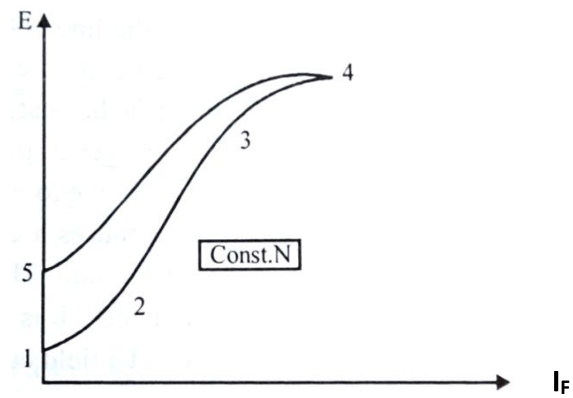



**Fig: No load magnetization curve (or OCC) of a DC Machine ( Plot of Armature Voltage $E_a$ Vs.Field current $I_F$)**

Though the field current is zero we get a small value of Armature voltage as seen at point 1 due to the residual magnetism present in the field coil. Subsequently armature voltage increases with field current upto some point 3 and then the rate of rise decreses. Finally at poin 4 field flux gets saturated and hence the emf also gets saturated. The plot of armature voltage vs.field current is not same during the field current reduction as that during the field current increase and this is due to the property of magnetic hysteresis in the Ferro magnetic materials. In the return path the induced volatage at zero field current is higher than that during the field current increase. This is due to the combined effect of Hysterisis and the residual magnetism.

## Different Types of DC Generators and their Terminal ( or Load ) Characteristics:

The DC generators are classified according to the manner in which the field flux is produced. Let us consider the following important types of DC Generators and their characteristics along with their equivalent circuits.

The following notation is used uniformly in all the following circuits/characteristics:

- $V_T$ = Generator's Terminal Voltage
- $I_L$ = Load or line current
- $I_A$ = Armature current

- **$E_A$** = Armature voltage
- **$R_A$ =** Armature Resistance
- **$I_F$** = Field current
- **$V_F$** = Field voltage
- **$R_F$ =** Field Resistance

**Separately Excited Generator:** In this type the field flux is derived from a separate power source which is independent of the Generator. The equivalent circuit of such a machine along with the governing equations is shown in the figure below.

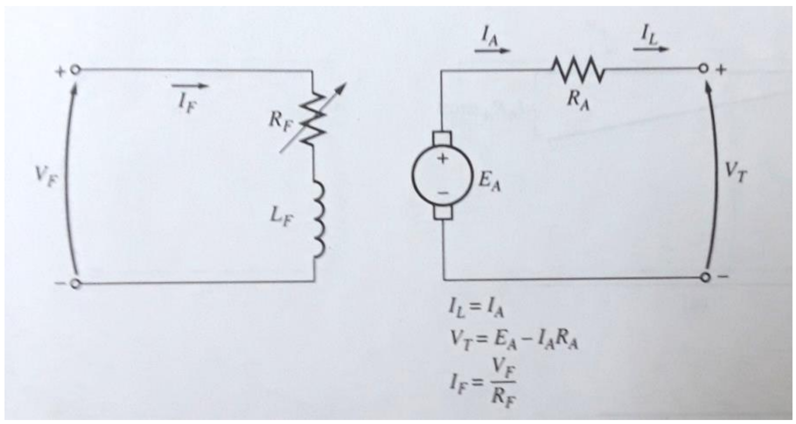



**Fig: Equivalent circuit of a separately excited DC Generator**

The terminal characteristic of this type of Generators is a plot of **$V_T$ vs. $I_L$** for a constant speed **ω** and the governing equations are :

- The Load or line current **$I_L$** = The armature current **$I_A$**
- Generator's Terminal Voltage = **$V_T$** = **($E_A - I_A R_A$)**
- **$I_F = V_F / R_F$**

Since the internally generated voltage is independent of **$I_A$**,the terminal characteristic of a Separately Excited Generator is a straight line as shown in the figure below.

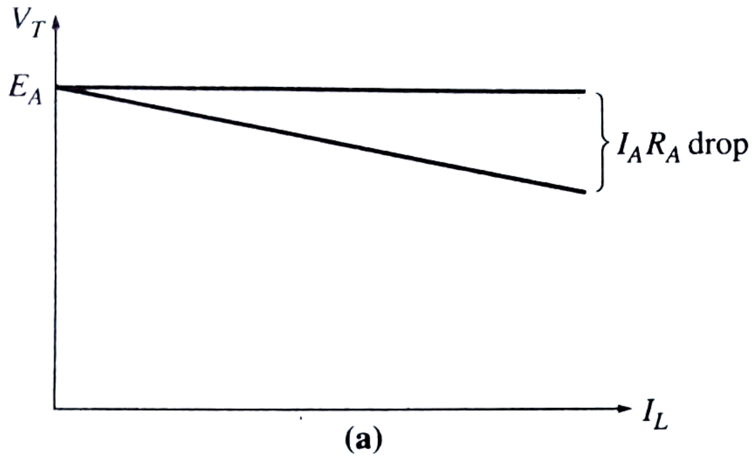
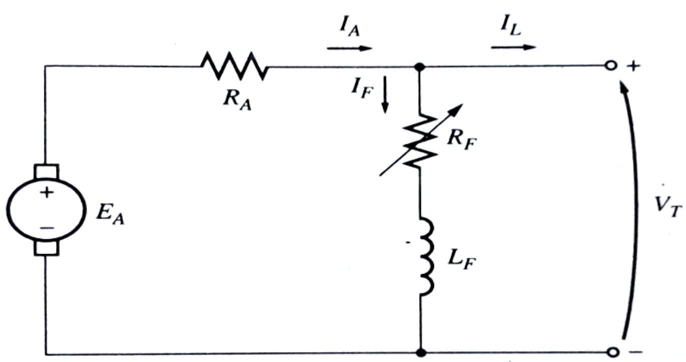



**Fig: The terminal Characteristics of a Separately Excited DC Generator**

When the load supplied by the generator increases, the load current $\mathbf{I_L}$ increases and hence the armature current $\mathbf{I_A}$ also increases. When the armature current increases, the $\mathbf{I_A R_A}$ drop increases, so the terminal voltage of the generator droops (falls). It is called a drooping characteristic.

**Shunt Generator:** In this the field flux is derived by connecting the Field directly across the Armature terminals. The equivalent circuit of such a generator is shown in the figure below along with the governing equations.




$$I_A = I_F + I_L$$
$$V_T = E_A - I_A R_A$$
$$I_F = \frac{V_T}{R_F}$$

**Fig: The equivalent circuit of a DC Shunt generator along with the relevant governing equations**

As could be seen, in this machine the armature current supplies both the load current and the field current. Using the Kirchhoff's voltage law the terminal voltage is seen to be same as that of a separately excited voltage i.e. $\mathbf{V_T = (E_A - I_A R_A)}$. In this the advantage is that no external supply is required for the field circuit. *But this leaves an important question. If the generator supplies its own field current how does it get the initial field flux that is required to start the machine and generate voltage when it is first turned on? This is explained below.*

## Build-up of E.M.F, Critical Field Resistance and Critical Speed :

### Voltage build up in a Shunt Generator :

The voltage build up in a shunt generator depends upon the presence of a ***residual flux*** in the poles of the generator. When a Shunt generator first starts to turn on an internal voltage is generated which is given by $\mathbf{E_A = k.\,\text{Ø}_{res}.\,\omega}$. This voltage( which may be just one or two volts ) appears at the generator terminals. This causes a current to flow in the generator's field coil $\mathbf{I_F = V_T / R_F}$. This produces a m.m.f. in the poles which in turn increases the flux in them. The increase in the flux causes an increase in $\mathbf{E_A = k.\text{Ø}\uparrow.\,\omega}$ which in turn increases the terminal voltage $\mathbf{V_T}$. When $\mathbf{V_T}$ rises, $\mathbf{I_F}$ increases further, increasing the flux more which increases $\mathbf{E_A}$ and so on. This voltage build up phenomenon is shown in the figure below.

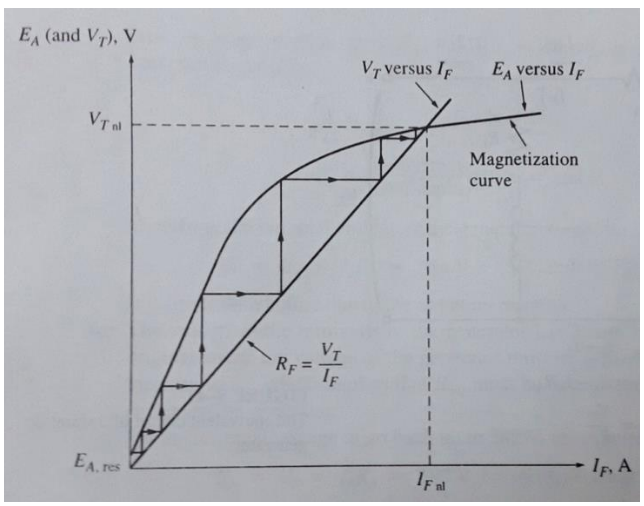



**Fig: Voltage build up on starting in a DC Shunt generator**

It is to be noted here that it is effect of ***magnetic saturation*** in the Pole faces which eventually limits the build of the terminal voltage.

The voltage build up in the figure above shows up as though it is building up in discrete steps. It is not so. These steps are shown just to make it clear the phenomenon of positive feedback between the Generator's internal voltage and the field current. In the DC Shunt generator both $\mathbf{E_A}$ and $\mathbf{I_F}$ increase simultaneously until the steady state conditions are reached.

**Critical Resistance:** For understanding the terms *critical Resistance* and *critical speed*, the open circuit characteristic (OCC) or the magnetization characteristic of a DC machine is shown again in the figure below along with *air gap* line and $R_f$ line. The extension of the liner portion of the magnetization curve, shown dotted in the figure below is known as the *air-gap line* as it represents mainly the magnetic behavior of the machine's air-gap.

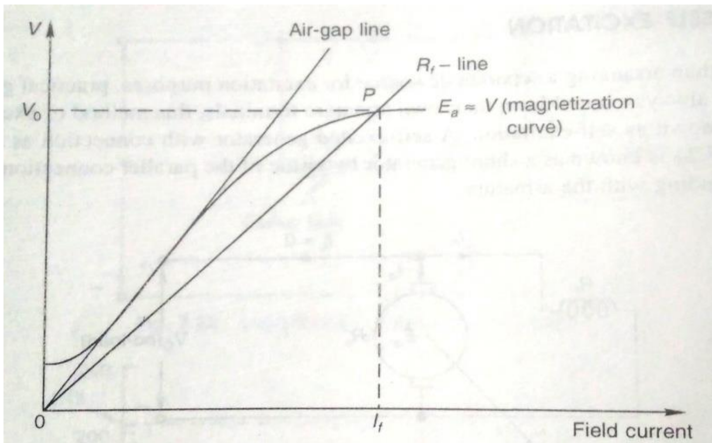



**Fig: Open Circuit Characteristic of DC machine along with Air gap and $R_f$ lines**

As already explained in the topic *Build up of EMF in a DC shunt generator:* At the instant of switching on the field after the armature has been brought to rated speed, the armature voltage corresponds to a small residual value which causes a small field current to flow. If the field is connected such that this current increases the field mmf and therefore the induced emf, the machine voltage cumulatively buildups and settles at a final steady value because of the saturation characteristic of the machine's magnetic circuit.

Since the generator is assumed to be on no-load during the build-up process, the following circuit relationships apply with reference to the machines' equivalent circuit shown in the figure below.

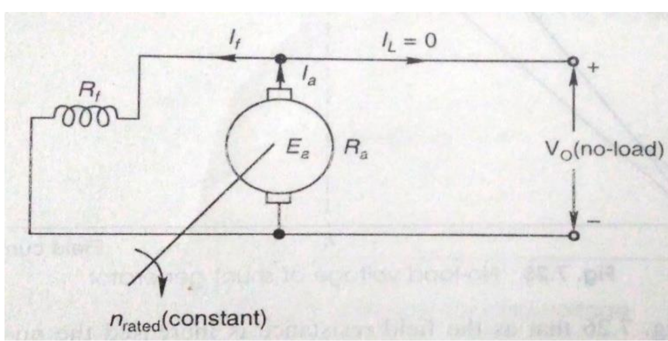



**Fig: The equivalent circuit of DC shunt generator**

$$I_a = I_f$$
$$V = E_a - I_f R_a$$

The field current in a shunt generator being very small, the voltage drop $I_f R_a$ can be neglected so that : $V_o = E_a(I_f)$ (magnetization characteristic)

And for the field circuit:

$$V_o = I_f\,R_f$$

which is a straight line relationship, called the $R_f$-line as shown in the OCC plot earlier. The no-load terminal voltage is the solution of the above two eqations for $V_o$.Thus the intersection point *P* of the $R_f$ -line with the magnetization characteristic as shown in the OCC gives the no-load terminal voltage ($V_0$) and the corresponding field current. Further, it is easy to visualize from this figure that the no-load voltage can be adjusted to a desired value by changing the field resistance.

It can be seen in the figure below that as the field resistance is increased the no-load voltage decreases.

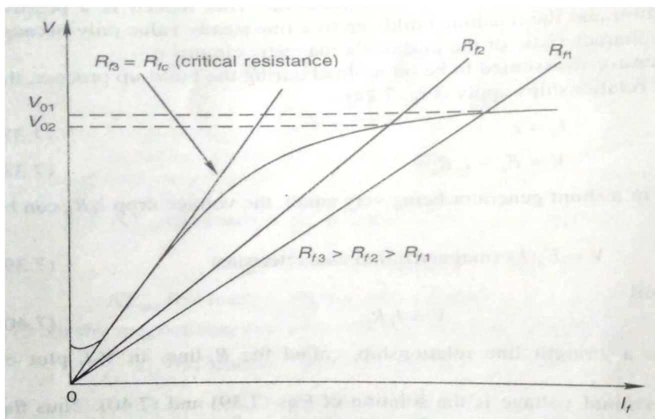



**Fig: Variation of No load voltage with field resistance**

The no-load voltage is undefined for a field resistance ($R_{f3} = R_{fc}$) whose line coincides with the linear portion of the magnetization curve. With field resistance even slightly more than this value, the machine does not excite to any appreciable value and would give no-load voltage close to the residual value. The machine with this much resistance in the field fails to excite and the corresponding resistance is known as the *critical resistance* (***$R_{fc}$***).

**Critical speed**: Consider now the operation with fixed ***$R_f$*** and variable armature speed as illustrated in the figure below. It can be observed that as the speed is reduced, the OCC proportionally slides downwards so that the no-load voltage *reduces.* At a particular speed, called the *critical speed,* the OCC becomes tangential to the $R_f$ line and as a result the generator would fail to excite.

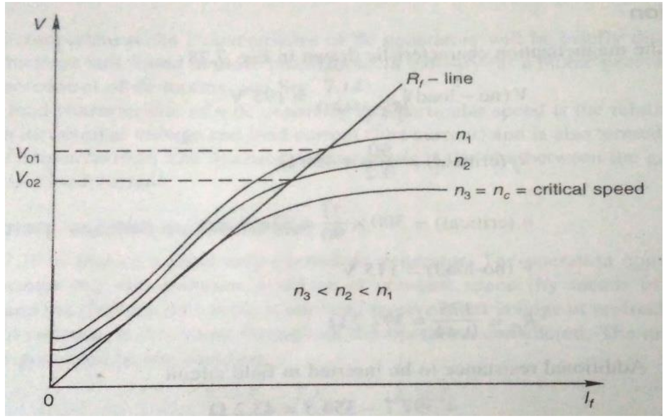



**Fig: Effect of speed on No load voltage**

**Causes of failure to excite in a Shunt Generator:**

A shunt generator may not get excited in certain conditions. The causes of such failure to excite, the method of detection and the corresponding remedial measures are given in the table below.

| S.No | Cause | Method of detection | Remedy |
|---|---|---|---|
| 1 | Absence of residual magnetism due to ageing | Zero reading on Voltmeter after rotating the machine | Operate the Generator as separately excited machine first and then as separately excited |
| 2 | Wrong field winding connections. Due to this the flux gets produced in opposite direction to that of the residual flux and they cancel each other. | Voltmeter reading decreases rather than increasing as the field current is increased | Interchange the field connections |
| 3 | Field resistance is more than the Critical field resistance. | Voltmeter shows zero reading | Field resistance to be reduced using suitable field diverter |
| 4 | Generator is driven in opposite direction | This wipes out the residual flux and the machine fails to excite | Generator to be driven in the proper direction |

**Terminal characteristics of a shunt generator :**

*The terminal characteristics* of the shunt generator differ from that of the separately excited generator because the amount of field current depends on its terminal voltage. As the generator load is increased, the load current $I_L$ increases and so $I_A = I_F + I_L\uparrow$ also increases . An increase in $I_A$ increases the $I_AR_A$ drop causing $V_T = (E_A - I_A\uparrow R_A)$ to decrease. This is precisely the same behavior we have seen in the case of separately excited generator. However, in the shunt generator when $V_T$ decreases the field current decreases ,hence the field flux deceases thus decreasing the generated Voltage $\mathbf{E_A}$ **.** Decreasing the $\mathbf{E_A}$ causes a further decrease in the terminal voltage $\mathbf{V_T = (E_A\downarrow - I_A R_A)}$**.** The resulting characteristic is shown in the figure below.

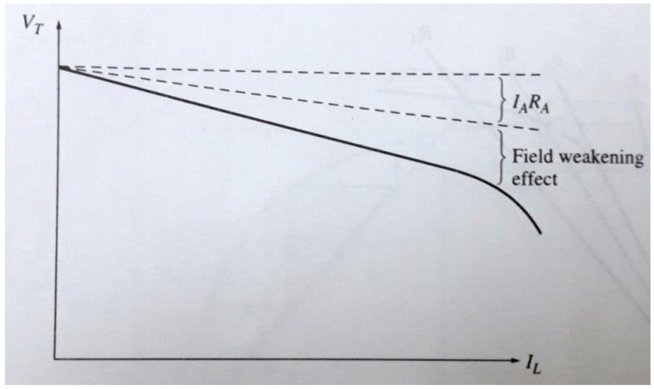
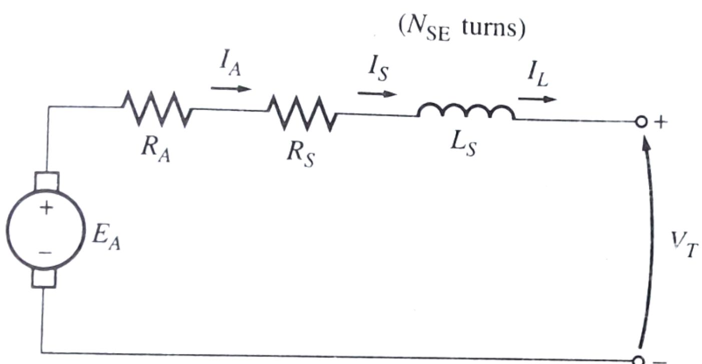



**Fig: Terminal Characteristic of DC Shunt Generator**

It can be noticed that the drop with load is steeper than that of a separately excited motor due to the field weakening affect. This means that the regulation of a Shunt Generator is worse than that of a Separately Excited Generator.

**DC Series Generator:** In this the field flux is derived by connecting the Field coil in series with the Armature of the Generator as shown in the figure below.




$$I_A = I_S = I_L$$
$$V_T = E_A - I_A(R_A + R_S)$$

**Fig: Equivalent circuit of DC Series Generator along with the governing equations**

As shown, the armature current, load current and field current are same in a DC series generator. i.e $\mathbf{I_A = I_F = I_L}$ . Since the mmf produced by the fields is given by $\mathcal{F}$, $\mathbf{= NI}$ and the field current is more in the DC series generator , the fileld winding is wound with lesser number of turns and also with a thicker gauge so as to offer less field resistance since full load current flows through the field winding .

The terminal characteristic of a DC Series Generator looks very much like the magnetization curve of any other type of generator and is shown in the figure below.

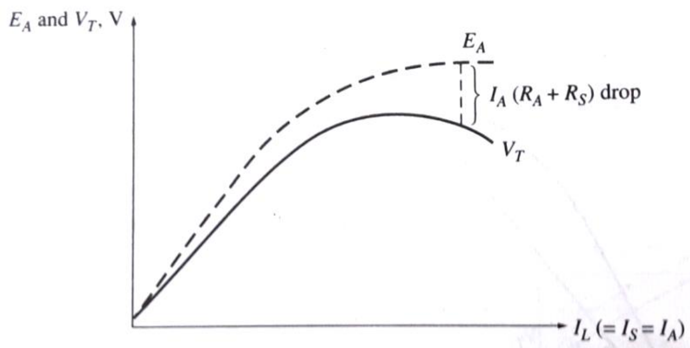



**Fig: Terminal Characteristic of DC Series Generator**

At no load however since there is no field current,armature voltage $\mathbf{E_A}$ and also the terminal voltage $\mathbf{V_T}$ are very small ( generated by the small amount of residual flux.) As the load increases ,field current rises hence $\mathbf{E_A}$ also increases rapidly. The $\mathbf{I_A (R_A+R_F)}$ drop also goes up but this rise is less predominant compared to the rise in $\mathbf{E_A}$ initaially and hence $\mathbf{V_T}$ also rises initially. After some time field flux gets saturated and hence the induced voltage $\mathbf{E_A}$ will be constant without any further rise. At this stage the resistive drop predominates and hence the ***terminal voltage $V_T$ starts drooping*.**

**DC Compound generator**:

As we know in DC shunt Generator the terminal Voltage falls and in a DC series generator the terminal voltage increases on loading. A compound DC Generator is the one in which there will be both Series and shunt field coils. If they are wound such that they aid each other then it is called a Cumulative Compound DC Generator and if they are wound such that the two fields oppose each other, then it is called a differential Compound DC Generator. The equivalent circuit diagram of such Cumulative DC Generator along with relevant governing equations is shown in the figure below.

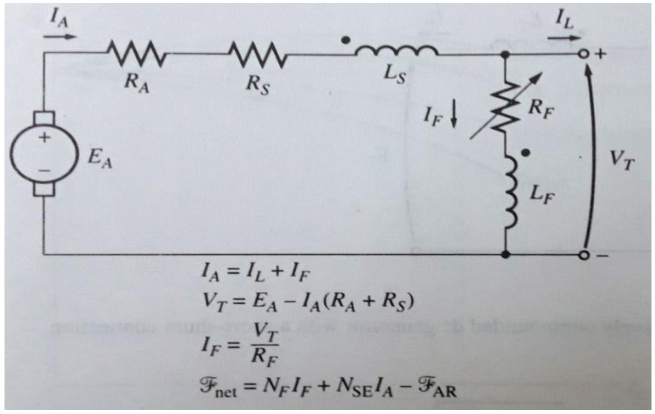



**Fig: Equivalent circuit of a Cumulative compound DC Generator**

The circuit diagram is shown with standard ***dot convention*** on the field windings.

***i.e. The current flowing into the dot side of the winding produces a positive mmf .***

And as can be seen that both $\mathbf{I_F}$ in the shunt winding and $\mathbf{I_A}$ in the series winding flow into the dot side and hence both produce magnetic fields which are positive and hence aid each other.

When the two fields are aiding each other we get a characteristic which will have the combined effect of ***drooping*** (due to the shunt coil) and ***rising*** (due to the field coil). Whichever coil current is more its effect will be more predominant. The terminal characteristics of a cumulative compound DC Generator are shown in the figure below for all the three cases.

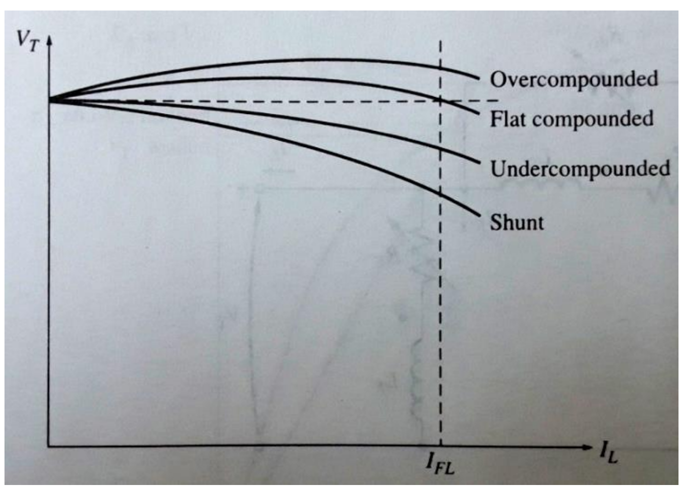



**Fig: Terminal Characteristics of a DC Compound Generator**

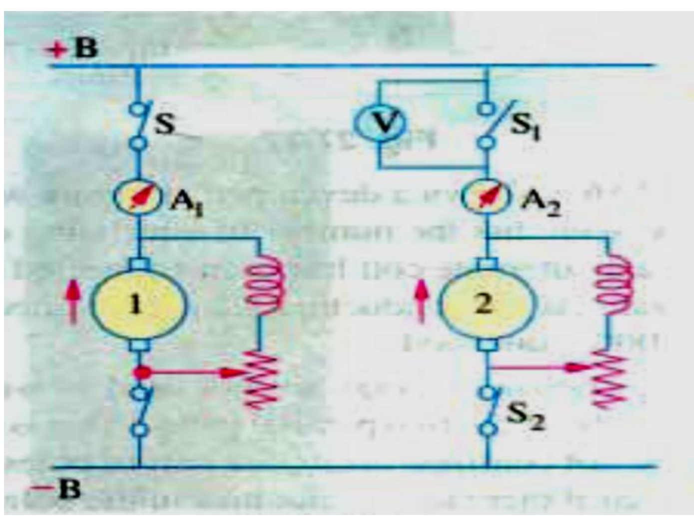
1. If the Series field effect is more dominating than that of the Shunt field coil then we get the ***Over compounded*** characteristic where the full load terminal voltage is higher than the no load terminal voltage.
2. If the Series field effect is equal to that of the Shunt field coil then we get the ***Flat compounded*** characteristic where the full load terminal voltage is equal to the no load terminal voltage.
3. If the Shunt field effect is more dominating than that of the Series field coil then we get the ***Under compounded*** characteristic where the full load terminal voltage is lower than the no load terminal voltage.

   The normal shunt characteristic is also shown in the figure for comparison.

## Parallel operation of DC Generators:

**Procedure for bringing in a second generator in Parallel with an existing Generator -1:**

The armature of generator -2 is speeded up to its rated value by the prime mover and then the switch S2 is closed. Thus with the voltmeter across the open switch S1 (which is called the circuit breaker) the circuit gets completed. The excitation of the incoming generator is adjusted so as to make the Voltmeter reading zero. This means that the terminal voltage is same as that of the generator-1 or the bus bar voltage. Under these conditions however the generator-2 is not supplying any load current because its induced e.m.f is same as that of the bus bar voltage and there can be no flow of current between terminals at the same potential. The generator-2 is said to be floating on the bus bar. The induced e.m.f of the generator -2 is increased by strengthening its field till it takes its proper share of the load. At the same time it may be necessary to weaken the field of generator -1 to maintain the bus bar voltage V constant.




**Summary Procedure for Paralleling of DC Shunt Generators:**

- Close the disconnect switch of the incoming Generator
- Start the prime mover and adjust it to the rated speed of the Machine
- Adjust the voltage of the incoming generator a few volts higher than the bus voltage
- Close the Circuit breaker of the incoming generator
- Turn the field rheostat of the incoming machine in the raise direction and that of the other machine (already on the bus ) in the lower voltage direction till the desired load distribution ( as indicated by the ammeters) is achieved.

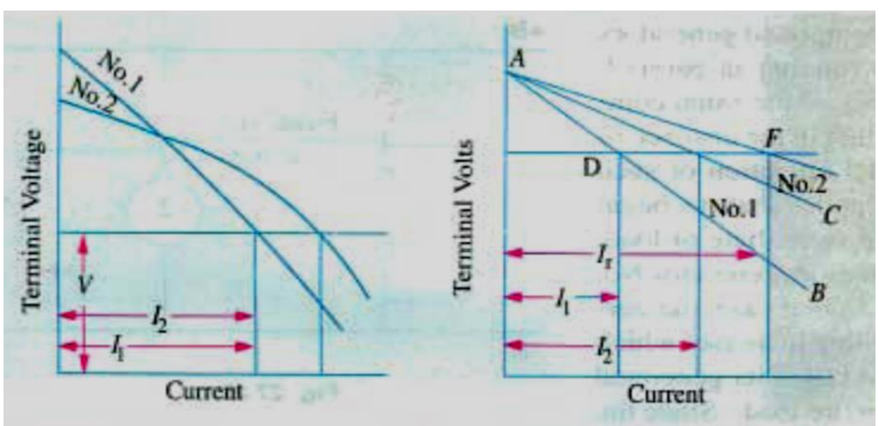
**Procedure for taking a Generator out of Parallel operation:**

For taking a generator out of parallel operation, its field is weakened and that of the other generator is strengthened gradually till the ammeter of the generator to be cleared reads zero. After that its circuit breaker and then the switch are opened sequentially thus removing the generator out of service. This method of connecting in and removing a generator from service helps in preventing any shock or sudden disturbance to the prime mover or to the system of parallel generators.

It is obvious that if the field of one generator is weakened too much, then power will be delivered to it and it will run in its original direction as a motor thus driving its prime mover.

**Parallel operation of Generators with different voltage characteristics:**

The voltage characteristics of two shunt generators with different regulation are shown in the figure below. It is seen that the Generator -1 delivers $\mathbf{I_1}$ amperes and Generator -2 delivers $\mathbf{I_2}$ amperes. It can be seen that the Generator having more drooping characteristic delivers lesser current. It can be seen that the two shunt generators will divide the load equally if their terminal characteristics are similar from no load to full load. i.e. their % regulation is same .




(a) (b)

**Figure: Voltage characteristics of two Generators with different % regulation**

When the individual characteristics of the generators are known, their combined characteristics can be drawn by adding the separate currents at a number of voltage points and plotting them against each other as shown in figure (b) above. From this combined characteristic, the voltage for any combined load can be read off and from there, the current supplied by each generator can be found out.

If the Generators have straight line characteristics, then the above result can be obtained by simple calculations instead of graphically.

**Load sharing of two generators with unequal no load voltages:**

Let us study the load sharing of two generators which have unequal no load voltages.

Let $E_1, E_2$ = no-load voltages of the two generators

$R_1, R_2$ = their armature resistances

$V$ = common terminal voltage

Then $I_1 = \frac{E_1 - V}{R_1}$ and $I_2 = \frac{E_2 - V}{R_2}$

$$\therefore \quad \frac{I_2}{I_1} = \frac{E_2 - V}{E_1 - V} \cdot \frac{R_1}{R_2} = \frac{K_2 N_2 \Phi_2 - V}{K_1 N_1 \Phi_1 - V} \cdot \frac{R_1}{R_2}$$

From the above equations, it is clear that the bus-bar voltage can be kept constant (and load can be changed from 1 to 2 ) by increasing $\phi_2$ or $N_2$ or by reducing $\phi_1$ or $N_1$. $N_1$ and N2 are changed by changing the speed of the prime movers and $\phi_1$ and $\phi_2$ are changed by using regulating shunt field resistances.

**Parallel operation of Generators with different Power Ratings :**

If it is desired that two generators with different power ratings when paralleled have to share the load in proportion to their ratings, then their external characteristics when plotted in terms of their % full load currents (not actual currents ) must be identical as shown in the figure below.

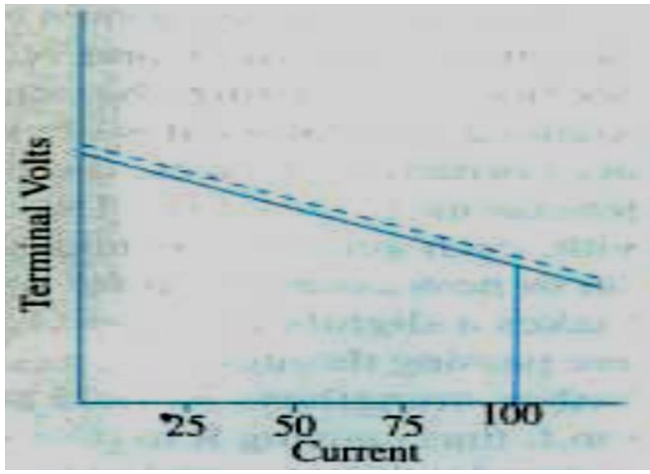



**Figure: Voltage characteristics of two Generators with same % regulation**

**( Here the current indicated is not of absolute value but it is the % of full load current )**

If for example a 100 kW generator is working in parallel with a 200 kW generator to supply a total of 240 kW, then first generator will supply 80kW and the second generator will supply the balance 160 kW

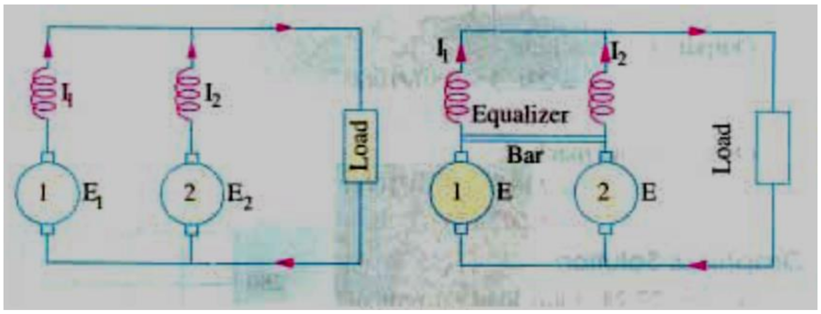
**Load sharing Principles:**

- Parallel shunt generators having equal no load voltages share the load in such a ratio that the load current in each machine produces the same armature voltage drop in each Generator. (In inverse proportion to their armature resistances )
- Parallel shunt generators having unequal no load voltages share the load in such a ratio that the load current in each machine produces the armature voltage drop in each Generator such that their terminal voltages are equal..
- The generator with the lowest drop assumes greater share of the change in the bus load
- Parallel Shunt Generators with different power ratings but same voltage regulation will share any oncoming bus load current in direct proportion to their power rating.

## Parallel Operation of Series Generators:

Figure (a) below shows two identical series Generators connected in series. Suppose $E_1$ and $E_2$ are equal, then the two generators supply equal currents and the system is stable. Let us now assume that for some reason due to a transient phenomenon, Generator -1 takes a slightly increased load. Then the current passing through the series field winding increases which in turn strengthens its field thus raising the generated e.m.f. causing it to take still higher load current. Since the given load current is fixed, the current supplied by generator -2 decreases thus decreasing it's field current and thus reducing it's e.m.f.

further. Since this effect is cumulative Generator-1 will ultimately supply the entire load current and eventually drives the Generator-2 as a motor. The circuit breaker of the Generator -1 will trip thus stopping their parallel operation.




(a) (b)

**Figure: Two Series Generators in Parallel (a) Without Equalizer Bar (b) With Equalizer Bar**

To avoid this condition and for making the parallel operation of series generators stable, they are always used with an equalizer bar connected to the armature ends of the series field coils of the generators as shown in figure(b). The equalizer bar is a solid thick conductor of very low resistance and its operation is as follows.

Suppose the generator -1 starts taking higher share of the load then its series field current increases. But now this increased current flows partly though its own field winding and partly though the equalizer bar &the series field winding of generator-2 . Hence the generators are affected in a similar manner and Generator -1 will not take the entire load.

For maintaining proper division of load from no load to full load it is essential that

- The regulation of each generator is same
- The series field resistances are inversely proportional to the generator ratings.

## Important concepts and Formulae:

- **Voltage generated in a DC machine: $E_A$ = (Ø ZN/60). (P/a) and in terms of angular speed ω :**
  **$E_A = K_a Ø ω$ where $K_a = ZP/2πa$**

## Illustrative Examples:

**Ex.1:** Calculate the e.m.f. generated by a 6 pole DC Generator having 480 conductors and driven at a speed of 1200 RPM. The flux per pole is 0.012 webers. (a) When the machine is lap wound (b) When the machine is wave wound

**Solution:** We know that the e.m.f. generated by a DC Generator is given by

$E_A = (\phi ZN/60)(P/a)$ where

**Ø** Flux per pole (webers) = 0.012Wb

**Z** Total number of conductors on the armature = 480

**a** The number of parallel paths **=** No of Poles P ( = 6 ) when Lap wound and = 2 when wave wound

**N** Speed of rotation of the machine (RPM) = 1200 RPM

**P** The number of poles = 6

(a) For Lap wound machine **a** = P = 6

$E_a$ = [ (0.012 x 480 x 1200 ) / 60 ] [ 6/6] **= 115.2 Volts**

(b) For wave wound machine **a** = 2

$E_a$ = [(0.012 x 480 x 1200 ) / 60 ] [ 6/2] **= 345.6 Volts**

**Ex.2 :** A 50 Kw ,250 V shunt generator operates at 1500 RPM .The armature has 6 poles and is lap wound with 200 turns. Find the induced e.m.f and the flux per pole at full load given that the armature and the field resistances are 0.01 Ω and 125 Ω respectively.

**Solution:**

Output line current = Output power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 125 = 2 A

Armature current in a shunt generator: $= I_l + I_f$ = 200 + 2 = 202 A

**Induced e.m.f $E_a$ :** = Line Voltage + Armature drop ($I_aR_a$ drop)

= 250 + 202 x 0.01 = **252.02 V**

But we know that armature voltage in terms of the basic machine parameters is also given by

**$E_A$ = (Ø ZN/60)( P/a)** where

**Ø** c: Flux per pole (webers) = To be determined

**Z** : Total number of conductors on the armature = Number of turns x 2 ( since each turn has two conductors ) = 200 x 2 = 400

**a** **:** The number of parallel paths **=** No of Poles P ( = 6 ) (since Lap wound )

**N** : Speed of rotation of the machine (RPM) = 1500 RPM

**P** **:** The number of poles = 6

∴ **Ø = ($E_A$ x 60 x a/ ZNP)** = 252.02 x 60 x 6 / 400 x 1500 x 6 = **0.025202 Wb**

**Ex.3:** A shunt generator connected in parallel to supply mains is delivering a power of 50 Kw at 250 V while running at 750 RPM. Suddenly its prime mover fails and the machine continues to run as a motor taking the same 50 Kw power from 250 V mains supply. Calculate the speed of the machine when running as a motor given that $R_a$ = 0.01 Ω, $R_f$ = 100 Ω and brush drop is 1 V per brush.

**Solution:**

***First let us calculate the Voltage generated by the machine while running as a generator under the given conditions:***

Output line current = Output power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 100 = 2.5 A

Armature current : $I_l + I_f$ = 200 + 2.5 = 202.5 A

Induced e.m.f $E_a$ : = Line Voltage + Armature drop ($I_aR_a$ drop)+ Brush drop(two brushes )

= 250 + 202.5 x 0.01 + 2 x1 = 254.025 V

***Next let us calculate the Voltage generated by the machine while running as a motor under the given conditions :***

Input line current = Input power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 100 = 2.5 A

Armature current : $I_l$ - $I_f$ = 200 - 2.5 = 197.5 A

Induced e.m.f or back e.m.f $E_b$ : = Line Voltage - Armature drop($I_aR_a$ drop) - Brush drop(two brushes )

= 250 – 197.5 x 0.01 - 2 x1 = 246.025 V

We know that the voltage induced in the machine is proportional to the speed i. e

Generator armature voltage is proportional to Generator speed : $E_a \propto N_G$ and similarly

Motor back e.m.f is proportional to Motor speed : $E_b \propto N_M$

Hence $E_a / N_G$ = $E_b / N_M$ or $\mathbf{N_M}$ = $(E_b / E_a) N_G$ = (246.025 / 254.025) x 750 = **726 RPM**

**Example 4 :** The following figures give the open-circuit characteristics of a dc shunt generator at 300 rpm:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $I_f$ (A) | 0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| $V_{oc}$ (V) | 7.5 | 93 | 135 | 165 | 186 | 202 | 215 |

The field resistance of the machine is adjusted to 354.5 W and the speed is 300 rpm.

(i) Determine graphically the no-load voltage.

(ii) Determine the critical field resistance.

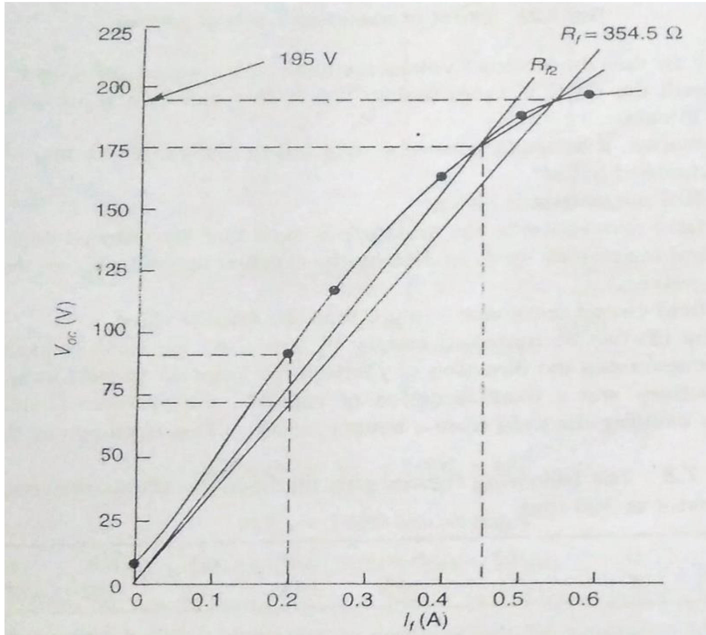
(iii) Determine the critical speed for the given field resistance.

(iv)What additional resistance must be inserted in the field circuit to reduce the no-load voltage to 175 V.

**Solution:**

Step-1 : Let us draw the Field resistance line corresponding to 354.5 Ω on the OCC ( magnetization characteristic). This can be done by identifying a point corresponding to a Voltage and current below the OCC corresponding to 354.5 Ω and extending the line joining this point with the origin.

1. $V_{No\ load}$ : This is the voltage corresponding to the point of intersection of the OCC and the $R_f$ line corresponding to 354.5 Ω and is seen to be = 195 V
2. $R_f$ critical : To obtain this draw a line tangential to the OCC starting from the Voltage at $I_f$ = 0 A. Note down the voltage at which the tangential deviates from the OCC and the corresponding $I_f$. Dividing this voltage by the corresponding $I_f$ we get the critical resistance viz. 90/0.2 = 450 Ω




3. Critical Speed: We know that as speed reduces the armature voltage reduces. i.e. the OCC leans down wards with decrease in speed and becomes tangential to the existing $R_f$ line itself. So to find out the critical speed we have to find out the new $E_a$ from the OCC corresponding to the lesser speed which deviates from the existing $R_f$ line. This is done by dropping a vertical perpendicular line from the point of deviation of the critical resistance line from the original OCC and identifying its intercept on the existing $R_f$ line. Then by drawing a line parallel to the $I_f$ axis from this point and locating its intercept with the Voltage axis, the new $E_a$ is found out.
   Then Critical speed = Original RPM x new $E_a$ / Original $E_a$ = 300 x71/90 = 236.7 RPM
4. To find out the additional resistance to be introduced into the field to get a new no load voltage of 175 V first we have to find out the value of $I_f$ corresponding to the new no load voltage. This can be directly read from the OCC and then from these voltage and current values we can directly get the new value of $R_f$ and thus the additional value of $R_f$ to be introduced into the field circuit.
   Thus new $R_f$ = 175/0.44 = 397.7 Ω and
   The additional resistance to be introduced into the field = 397.7 – 354.5 = 43.2 Ω

**Example 5:** Two 220 V DC generators each having linear external characteristics, operate in parallel. One machine has a terminal voltage of 270 V on no load and 220 V at load current of 35 A , while the other has a voltage of 280 V at no load and 220 V at 50 A . Calculate the output current of each machine and the bus bar voltage when the total load current is 60 A. What is the kW output of each machine under this condition?

Armature resistance of G-1 = $R_{a1}$ = ( 270 – 220 )/35 = 10/7 Ω

Armature resistance of G-2 = $R_{a2}$ = ( 280 – 220 )/ 50 = 1.2 Ω

Let the currents supplied by the two generators be $I_{L1}$ and $I_{L2}$ when the full load current is 60 A. Then from the armature circuit governing equations of the two generators we have:

Terminal Voltage V = $(E_{A1} - I_{L1}R_{A1})$ = 270 – 10/7 x $I_{L1}$ or

= $(E_{A1} - I_{L1}R_{A1})$ = 280 – 1.2 x $I_{L2}$

i.e. 270 – 10/7 x $I_{L1}$ = 280 – 1.2 x $I_{L2}$ or

Or after simplification 4.2 $I_{L2}$ – 5$I_{L1}$ = 35 A

And $I_{L1}$ + $I_{L2}$ = 60 A

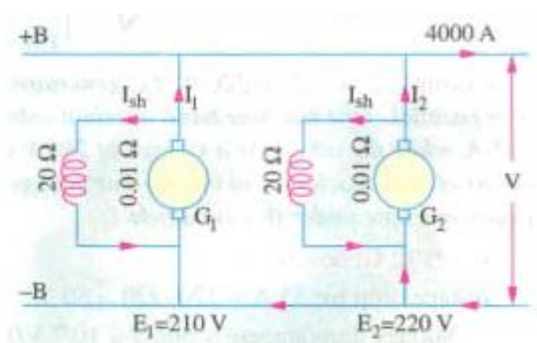
Solving these two equations we get : $I_{L1}$ = 22.8 A and $I_{L2}$ = 37.2 A

Terminal voltage = 270 – 10/7 x 22.777 = 237.4 V

Or = 280 – 1.2 x 37.222 = 237.4 V

**Example 6:** Two shunt generators each of an armature resistance of 0.01 Ω and field resistance of 20 Ω run in parallel and supply a total load current of 4000 Amps. The e.m.fs are respectively 210 V and 220 V. Calculate the bus bar voltage and output of each machine.

**Solution:** The two generators connected in parallel along with all the given data are shown in the figure below.




From the figure we have

$V$ = Bus bar voltage

$I_1$ and $I_2$ = Load currents supplied by generators 1 and 2 respectively.

$I_{sh1} = I_{sh2} = I_{sh} = V/20$ amps ( Field currents )

$I_{a1}$ = Armaturte current of machine-1 = $I_1 + I_{sh} = I_1 + V/20$ and similarly

$I_{a2}$ = Armaturte current of machine-2 = $I_2 + I_{sh} = I_2 + V/20$

We know that in each machine the armature induced voltage is given by :

$E_1 = V + I_{a1}. R_{a1} = V + (I_1 + V/20) ) \times 0.01 = 210$ V ( for first machine ) and

$E_2 = V + I_{a2}. R_{a2} = V + (I_2 + V/20) ) \times 0.01 = 220$ V ( for second machine )

Subtracting the eqn-2 from eqn-1 we get $0.01 (I_1 - I_2) = -10$ or

$(I_2 - I_1) = 1000$ Amps and we also have

$(I_2 + I_1) = 4000$ Amps

Solving the two equations we get $I_1 = 1500$ Amps and $I_2 = 2500$ Amps

Using this value of $I_1$ in the above expression for $E_1$ we get

$V + (1500 + V/20) ) \times 0.01 = 210$ V

***Solving this equation we get the bus bar voltage V = 194.9 Volts***

***Output power from 1$^{st}$ Generator = 194.9 x 1500/1000 = 292.35 kW***

***Output power from 2$^{nd}$ Generator = 194.9 x 2500/1000 = 487.25 kW***

**Example 7:** Two shunt generators operating in parallel deliver a total current of 250 A. One of the generators is rated 50 kW and the other 100 kW. The voltage rating of both machines is 500 V. Their regulations are 6% (smaller one) and 4 %. Assuming linear characteristics determine (a) the current delivered by each machine (b) the terminal voltage.

Machine -1 (50 kW, 6%, 500 V) : Full load current $I_{FL1}$= 50 x 1000 / 500 = 100 A

Full load voltage drop = 500 x 6 /100 = 30 V

Armature resistance $R_{a1}$ = Full load voltage drop / Full load current $I_{L1}$ = 30/100 = 0.3 Ω

Machine -2 (100 kW, 4%, 500 V) : Full load current $I_{FL2}$= 100 x 1000 / 500 = 200 A

Full load voltage drop = 500 x 4 /100 = 20 V

Armature resistance $R_{a2}$ = Full load voltage drop / Full load current $I_{L2}$ = 20/200 = 0.1 Ω

Let $I_1$ and $I_2$ be the load currents shared by the two generators respectively. Then the terminal voltage V is given by the two equations: $V = 500 - 0.3\ I_1$

$V = 500 - 0.1\ I_2$

Thus we have $0.3\ I_1 = 0.1\ I_2$ or $3\ I_1 = I_2$. Also $I_1 + I_2 = 250$ A . Solving these two equations we get:

$I_1$ = 6 2.5 A and $I_2$ = 187.5 A

(b) Terminal voltage: V = 500 – 0.1 x 187.5 = 481.25 V

**Example 8:** Two DC shunt generators A and B are connected in parallel and their load characteristics are linear (Straight lines). The voltage of generator -A falls from 240 V at no load to 220 V at 200 A and that of Generator-B falls from 245 V at no load to 220 V at 150 A. Determine the currents supplied by each machine to a common load of 300 A and the bus bar Voltage.

Armature resistance of G-1 = $R_{a1}$ = ( 240 – 220 )/200 = 0.1 Ω

Armature resistance of G-2 = $R_{a2}$ = ( 245 – 220 )/150 = 0.17 Ω

Let the currents supplied by the two generators be $I_{L1}$ and $I_{L2}$ when the full load current is 300 A. Then from the armature circuit governing equations of the two generators we have:

Terminal Voltage V = $(E_{A1} - I_{L1}R_{A1})$ = 240 – 0. 1 x $I_{L1}$ or

= $(E_{A1} - I_{L1}R_{A1})$ = 245 – 0.17 x $I_{L2}$ and $I_{L1} + I_{L2}$ = 300

Solving the two equations we get $I_{L1}$ = 170.4 A and $I_{L2}$ = 129.6 A

Terminal voltage = 240 – 0. 1 x 170.4 = 223 V

Or = 245 – 0.17 x 129.6 = 223 V

**Example 9:** Two DC shunt generators are connected in parallel to supply a load current of 1500A. One generator has an armature resistance of 0.5 Ω and an e.m.f. of 400 V while the other one has an armature resistance of 0.4 Ω and e.m.f. of 440 V. The resistances of the shunt fields are 100 Ω and 80 Ω respectively. Calculate the currents $I_1$ and $I_2$ supplied by the individual generators and the terminal voltage V of the combination.

**Solution:** The generator connection diagram is shown in the figure below.

**Let** **V** = Bus bar voltage (terminal voltage of the two generators in combination )

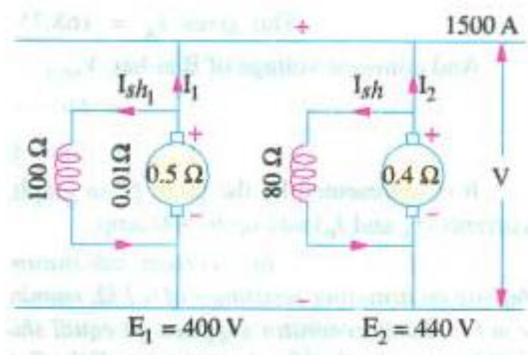
$I_1$ = Out put current of generator 1 and

$I_2$ = Out put current of generator 2

Then we have $I_2 = 1500 - I_1$

From the diagram below we can see that the field currents are: $I_{sh1} = V/100$ A and $I_{sh2} = V/80$ A

From the diagram we can also see that the load currents are : $I_{a1} = I_1 + I_{sh1}$ and $I_{a2} = I_2 + I_{sh2}$




Substituting the above field current values into the values of the two load currents we get:

$$I_{a1} = I_1 + V/100 \quad \text{and} \quad I_{a2} = I_2 + V/80$$

Substituting the value of $I_2$ in terms of $I_1$ we get: $I_{a2} = (1500 - I_1 + V/80)$

For each machine the terminal voltage is given by: **V = (Armature voltage – Armature drop)**

**Thus for machine -1 :** $V = 400 - (I_1 + V/100) \times 0.5$

$= 400 - 0.5I_1 - 0.005V$ or

$0.5I_1 = 400 - 1.005V$ .......... (1)

**And for machine -2 :** $V = 440 - (1500 - I_1 + V/80) \times 0.04$

$= 440 - 60 + 0.04\,I_1 - 0.0005V$ or

$0.04I_1 = 1.005V - 380$ .......... (2)

Dividing eqn.1 by eqn. 2 we get:

$0.5I_1 / 0.04I_1 = (400 - 1.005V) / (1.005V - 380)$

**i.e.** $12.5\,(1.005V - 380) = (400 - 1.005V)$ or $13.5675\,V = 5150$

**Thus V = 5150/13.5675 = 379.58 Volts**

**Substituting this value of V in the eqn. 1 above we get : $0.5I_1$ = 400 – 1.005 x 379.6**

**Or $I_1$ = 37 A and $I_2$ = 1500 – 37 = 1463 A**

**Out put power of Generator-1 = 379.58 x 37 /1000 kW = 14 kW**

**Out put power of Generator-2 = 379.58 x 1463 /1000 kW = 555 kW**

# UNIT – IV

## D.C. Motors & Speed Control Methods:

- D.C Motors
- Principle of operation
- Back E.M.F
- Torque equation
- Characteristics and application of Shunt, Series and Compound motors
- Armature reaction and commutation.
- Speed control of DC Motors
    - Armature voltage and field flux control methods
    - Ward- Leonard system
- Principle of 3 point and 4 point starters
- Protective devices.
    - **Important concepts and Formulae**
    - **Illustrative examples**

## DC Motors:

**Principle of operation:** DC Motors are DC machines used as motors. A DC Motor converts the input DC power into output rotational mechanical power from the following principle. A current carrying conductor placed in a magnetic field experiences a mechanical force given by **F= i (l X B).**

When a group of such conductors is placed on a rotor and are connected properly the force experienced by the all the conductors together gets translated into a torque on the rotor (armature) and it starts rotating. We will derive an expression for such a Torque developed by a DC Motor from the first principles and its equivalent circuit by equating the Electrical power given to the motor (excluding the losses) to the mechanical power developed by the motor.

### Torque developed by a DC Motor:

Consider the equivalent circuit of a DC motor as shown in the figure below.

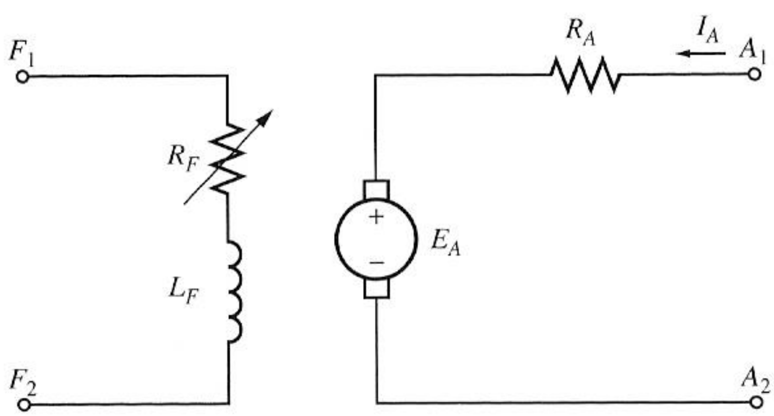



**Fig: Equivalent circuit of a DC motor**

In this figure, the armature circuit is represented by an ideal voltage source $\mathbf{E_A}$ and the armature resistance $\mathbf{R_A}$. The field coils, which produce the magnetic flux in the motor, are represented by inductor $\mathbf{L_F}$ and the field resistance $\mathbf{R_F}$. The separate external variable resistor used to control the amount of current in the field circuit is also combined with the field resistance and is together shown as $\mathbf{R_F}$.

We know from the earlier study of generators that the voltage generated in a DC Machine when It's armature is rotating in a magnetic flux of **Ø** webers/pole is given by $\mathbf{E_A = K_A. Ø.\omega}$ where $\mathbf{K_A}$ is given by:

$$\mathbf{K_A = (ZP/2\pi a)}$$

Now in the DC Motor also, when it is rotating, from the same fundamental principle of Generator a Voltage is generated across the armature and it is now called back EMF and is normally shown as $\mathbf{E_b}$ to distinguish it from the voltage generated in the armature of a generator which was shown as $\mathbf{E_A}$ **.**

The governing equation of the DC Motor armature circuit now becomes:

$$V_T = E_b + I_a R_A \quad \text{or} \quad E_b = V_T - I_a R_A$$

(as against $V_T = E_A - I_a R_A$ in the case of a generator where $I_A$ flows from armature towards the external terminals i.e external load )

Since now an external voltage $V_T$ is applied to the motor terminals , direction of armature current changes i.e. now it flows from external terminals towards the armature.

The power delivered to the motor is given by : $P_{in} = V_T . I_a$ . From this, the loss of power in the armature is equal to $I_a^2 R_A$ and hence the net power given to the motor armature is given by :

$$P_m = V_T . I_a - I_a^2 R_A = I_a (V_T - I_a R_A) = I_a . E_b$$

$$P_m = I_a . E_b$$

This net electrical power is converted into mechanical power. We know that in mechanical rotational systems the power is equal to Torque times the speed. In the SI system of units which is the present Industry standard it is given by :

$$P_{mech} \text{ (watts )} = \tau \text{ (Nw.mtrs )}.\omega \text{ (Radians/second )}$$

For simplification if we ignore the mechanical losses in the motor,then :

$$P_m = I_a . E_b = P_{mech} = \tau .\omega$$

$$\text{i.e. } \tau .\omega = I_a . E_b = E_b . I_a$$

Substituting the value of $E_A = K_A . \text{Ø} . \omega$ we got in generators here for $E_b$ since they are the same induced emfs we get $\tau .\omega = I_a . K_A. \text{Ø}.\omega$ or

$$\tau = K_A. \text{Ø} . I_a$$

It is to be noted that this expression for the torque induced in a motor is similar to the voltage induced in a DC Generator except that the speed $\omega$ in the DC Generator is replaced by the Armature current $I_a$ . The constant $K_A$ is same and is given by $K_A = (ZP/2\pi a)$

***In general, the torque τ in the DC motor will depend on the following 3 factors:***

1. ***The flux Ø in the machine***
2. ***The armature current $I_a$ in the machine***
3. ***The same constant $K_A$ representing the construction of the machine***

**Types of DC Motors and their output (or terminal) Characteristics:**

There are three important types DC Motors: DC separately excited, Shunt and Series motors. We will explain their important features and characteristics briefly.

The terminal characteristic of a machine is a plot of the machine's output quantities versus each other. For a motor, the output quantities are shaft ***torque*** and ***speed,*** so the terminal characteristic of a motor is a plot of its output ***torque versus speed.*** (Torque/Speed characteristics)

They can be obtained from the Motor's Induced voltage and torque equations we have derived earlier plus the Kirchhoff's voltage law around the armature circuit and are again given below for quick reference.

- The internal voltage generated in a DC motor is given by: $\mathbf{E_b = K_a. \Phi.\omega}$
- The internal Torque generated in a DC motor is given by: $\mathbf{\tau = K_a. \Phi.I_a}$
- KVL around the armature circuit is given by : $\mathbf{V_T = E_b + I_a.R_a}$

Where $\Phi$ = Flux per pole .... Webers
$I_a$ = Armature current .... Amperes
$V_T$ = Applied terminal Voltage .... Volts
$R_a$ = Armature resistance .... Ohms
$\omega$ = Motor speed .... Radians/sec
$E_b$ = Armature Back EMF .... Volts
$K_a$ = **(ZP/2πa)** : Motor Back EMF/Torque constant

From the above three equations we get the relation between Torque and speed as:

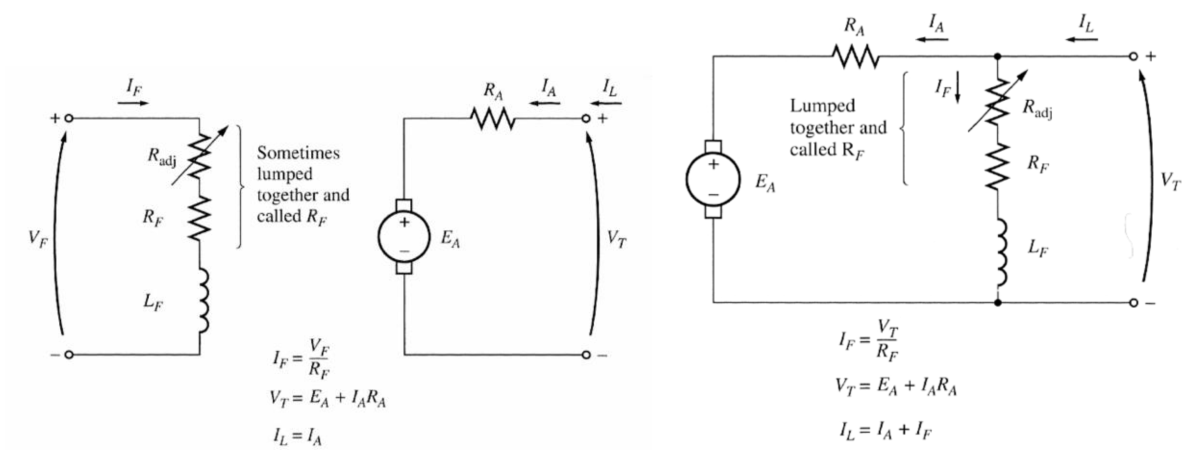
$$\omega = (V_T / K_a. \Phi) -- (R_a / K_a. \Phi). I_a$$

$$\omega = (V_T / K_a. \Phi) -- [R_a / (K_a. \Phi)^2].\tau$$

We will use this generalized equation in different types of motors and obtain their ***Torque vs. Speed*** characteristics.

**DC separately excited and Shunt Motors:**

The Equivalent circuits of DC separately excited and Shunt Motors along with their governing equations are shown in the figure below.




(a) Separately Excited (b) Shunt

**Fig: Equivalent circuit of DC separately excited and Shunt Motors**

In a separately excited DC motor the field and armature are connected to separate voltage sources and can be controlled independently. In a shunt motor the field and the armature are connected to the same source and cannot be controlled independently. When the supply voltage to a motor is assumed constant and is same to the field and armature circuits, there is no practical difference in behavior between these two machines. Unless otherwise specified, whenever the behavior of a shunt motor is described, it would be same as that of a separately excited motor.

In both their cases, with a constant field current the field flux can be assumed to be constant and then **($K_a.\Phi$)** Would be another constant **K**. Then the above Generalized Torque speed relations would become:

$$\omega = V_T / K -- (R_a/ K).\ I_a$$

Substituting the value of $I_a$ in terms of $\tau$ ($I_a = \tau / K_a.\Phi = \tau / K$) we get

$$\omega = V_T / K -- [R_a/ (K)^2].\tau$$

This equation is just a straight line with negative slope. The resulting Speed/ Torque Characteristics of a DC Separately Excited /Shunt Motor for a rated terminal voltage and full field current are shown in the figure below. It is a drooping straight line.

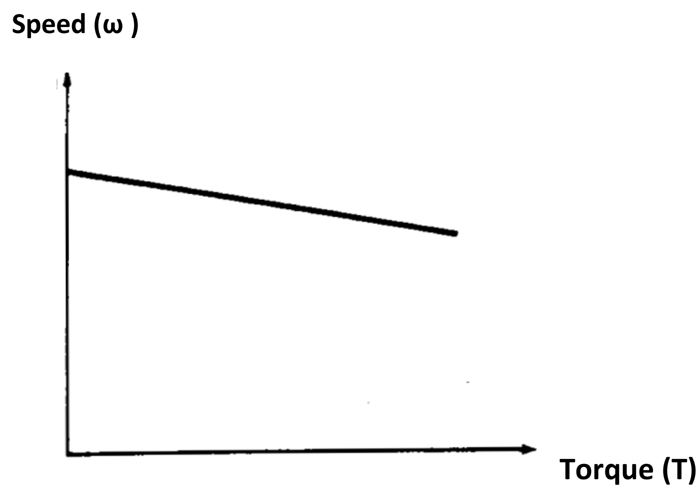



**Fig: Speed/ Torque Characteristics of a DC Separately Excited/Shunt Motor**

The no load speed is given by the Applied armature terminal voltage and the field current. Speed falls with increasing load torque. The speed regulation depends on the Armature circuit resistance. The usual drop from no load to full load in the case of a medium sized motor will be around 5%. Separately excited motors are mostly used in applications where good speed regulation and adjustable speed are required.

If motor armature reaction is taken into account, then as its load increases, the flux-weakening effects reduce its flux. From the motor speed equation above, the effect of reduction in flux is to increase the motor's speed at any given load over the speed it would run at without armature reaction. Though at a first glance of the Speed torque equation it may appear that the effect of reduction in flux is to decrease the motor's speed at any given load (since $\Phi^2$ is in the denominator**)** actually since the first positive term contains $V_T$ which is much larger quantity compared to the second negative term, IaRa drop the net effect would only be to increase the motor's speed at any given load. The torque-speed characteristic of a shunt motor with armature reaction is shown below:

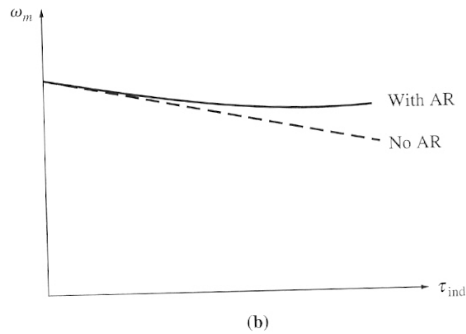



**Fig: Torque-speed characteristic of the motor with armature reaction considered**

**Motor's Other Characteristics:** Though the terminal characteristics **(Speed vs. Torque)** are only important for analysis of a DC motor performance, study and understanding of the other characteristics like speed vs. $I_a$ and Torque vs. $I_a$ would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

- **Speed vs.** $I_a$ : $E_b = K_a.\Phi.\omega = V_T - I_a.R_a$

  $\omega = (V_T - I_a.R_a)/ K_a.\Phi$

- **Torque vs. $I_a$**: $\tau = K_a.\Phi.I_a$

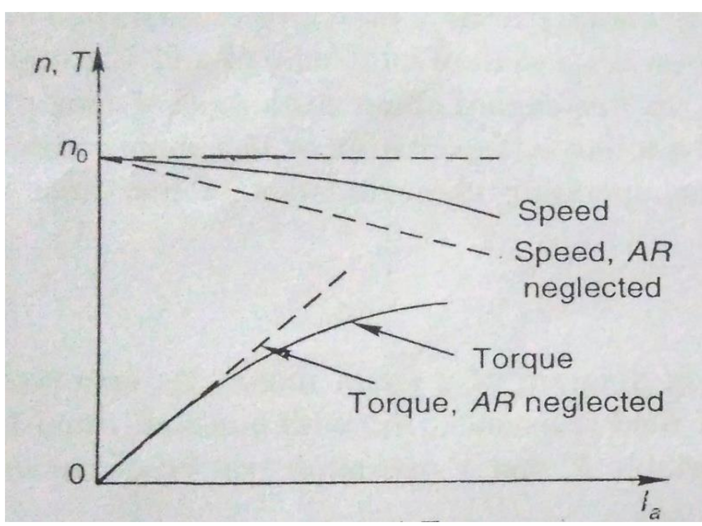



**Fig: Speed and torque vs. Armature current for a DC shunt motor**

**DC Series Motor:**

The equivalent circuit of a DC Series motor is shown in the figure below.

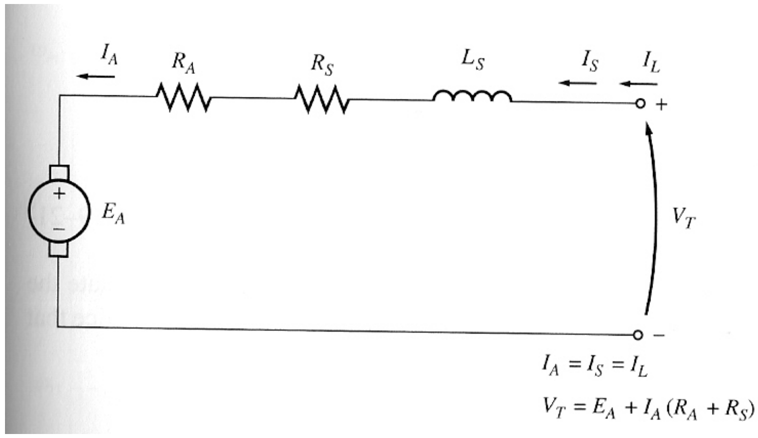



**Fig: Equivalent Circuit of a DC Series Motor**

In a series motor the field current and armature current are same and hence the field flux is directly dependent on the armature current. Hence during the initial i.e unsaturated region of the magnetization characteristic the flux **Φ** can be assumed to be proportional to the armature current.

Then $\mathbf{\Phi = K_f.I_a}$

And using this value in the first basic motor relation given earlier we get:

$$\tau = K_a.\ \Phi.I_a = K_a.\ K_f.I_a^2$$

$$\tau = K_{af}.I_a^2 \quad (\text{ where } K_{af} = K_a.K_f)$$

Substituting the above two values of **Φ** and **τ** in the second basic motor equation

$$\omega = (V_T / K_a.\ \Phi) -- [R_a/ (K_a.\ \Phi)^2].\tau$$

We get

$$\omega = V_T / K_a.\ K_f.I_a -- [R_a/ (K_a.\ K_f .I_a)^2].K_{af}.I_a^2$$

$$\omega = V_T / K_{af}.I_a -- [R_a/ (K_{af} .I_a)^2].K_{af}.I_a^2$$

$$\omega = V_T / K_{af}.I_a -- [R_a/ (K_{af})]$$

From the relation $\mathbf{\tau = K_a.\ \Phi.I_a = K_a.\ K_f.I_a^2}$ **we get** $\mathbf{I_a = \sqrt{\tau/K_{af}}}$ and substituting this in the above equation $\mathbf{\omega = V_T / K_{af}.I_a -- [R_a/ (K_{af})]}$

**We get**

$$\omega = [V_T /\sqrt{( K_{af}.\tau)}] -- [R_a/(K_{af})]$$

Where $R_a$ is now the sum of armature and field winding resistances and $K_{af} = K_a.K_f$ is the total motor constant. The Speed-Torque characteristics of a DC series motor as obtained from the above relation are shown in the figure below.

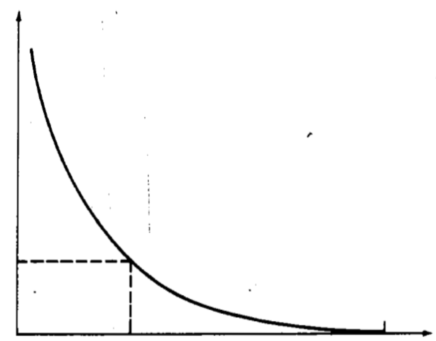



**Fig: Speed-Torque characteristics of a DC series motor**

**Motor's Other Characteristics:** Though the terminal characteristics **(Speed vs. Torque)** are only important for analysis of a DC motor performance, study and understanding of the other characteristics like speed vs. $I_a$ and Torque vs. $I_a$ would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

- **Speed vs.** $I_a$ :

$$E_b = K_a.\ K_f.\ I_a\ \omega = V_T - I_a.R_a$$

$$\text{i.e.} = K_{af}.\ I_a\ .\omega = V_T - I_a.R_a \quad \text{and}$$

$$\omega = (V_T - I_a.R_a) / K_{af}.\ I_a = (V_T / K_{af}.\ I_a) - (R_a / K_{af})$$

  This is an inverse relationship and is shown plotted in the figure below.

- **Torque vs.** $I_a$:

$$\tau = K_a.\ \Phi.I_a = K_{af}.I_a^2$$

  This is a direct relationship and is shown plotted in the figure below.

Saturation and armature reaction demagnetization both cause the flux per pole to increase (with respect to $I_a$ ) at a rate slower than the assumed linear relationship. Actual characteristics are shown in dotted lines.

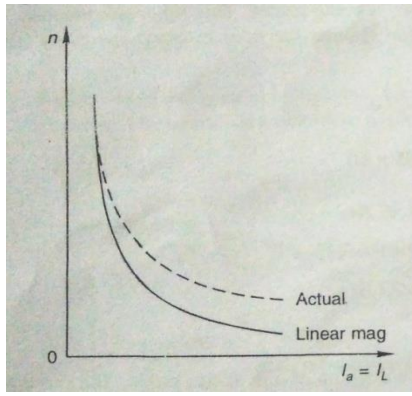



**Fig: Speed Vs. Armature current in a Series Motor**

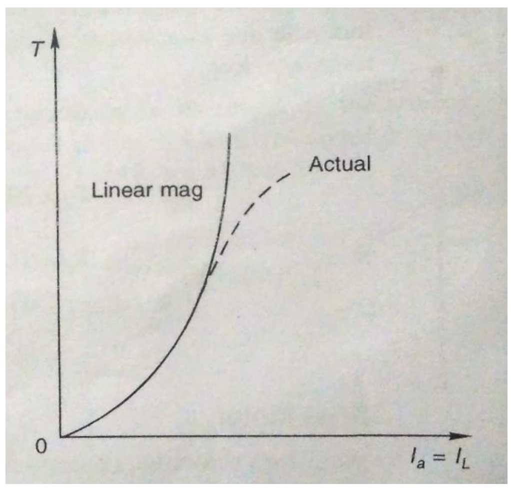


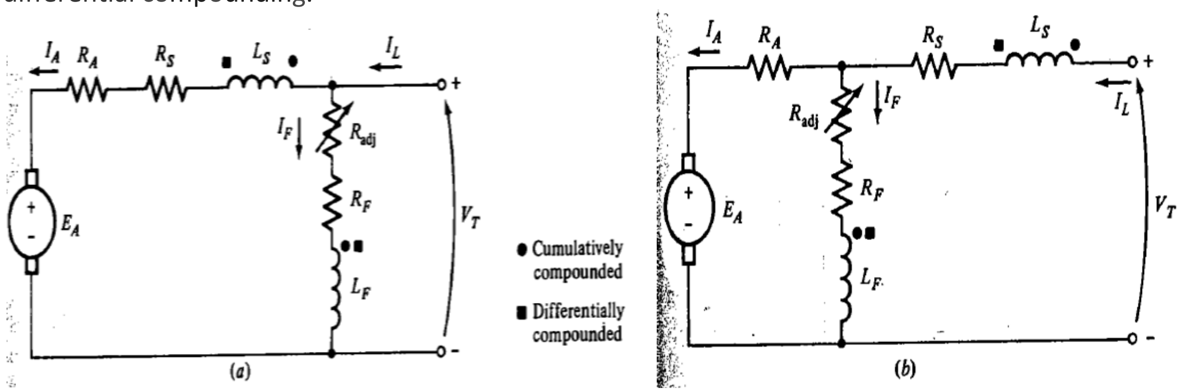

**Fig: Torque vs. Armature current in a DC Series Motor**

Series motors are suitable for applications requiring high starting torque and heavy overloads. Since Torque is proportional to square of the armature current, for a given increase in load torque the increase in armature current is less in case of series motor as compared to a separately excited motor where torque is proportional to only armature current. Thus during heavy overloads power overload on the source power and thermal overload on the motor are kept limited to reasonable small values. According to the above Speed torque equation, as speed varies inversely to the square root of the Load torque, the motor runs at a large speed at light load. Generally the electrical machine's mechanical strength permits their operation up to about twice their rated speed. Hence the series motors should not be used in such drives where there is a possibility for the torque to drop down to such an extent that the speed exceeds twice the rated speed.

**DC Compound Motor:**

A compound motor is a motor with *both a shunt and a series field.* Such a motor is shown in the Figure below. The dots that appear on the two field coils have the same meaning as the dots on a transformer: *Current flowing into a dot produces a positive magneto motive force.* If current flows into the dots on both field coils, the resulting magneto motive forces add to produce a larger total magneto motive force.

This situation is known as *cumulative compounding.* If current flows into the dot on one field coil and out of the dot on the other field coil , the resulting magneto motive forces subtract. In the Figure below the round dots correspond to cumulative compounding of the motor, and the squares correspond to differential compounding.



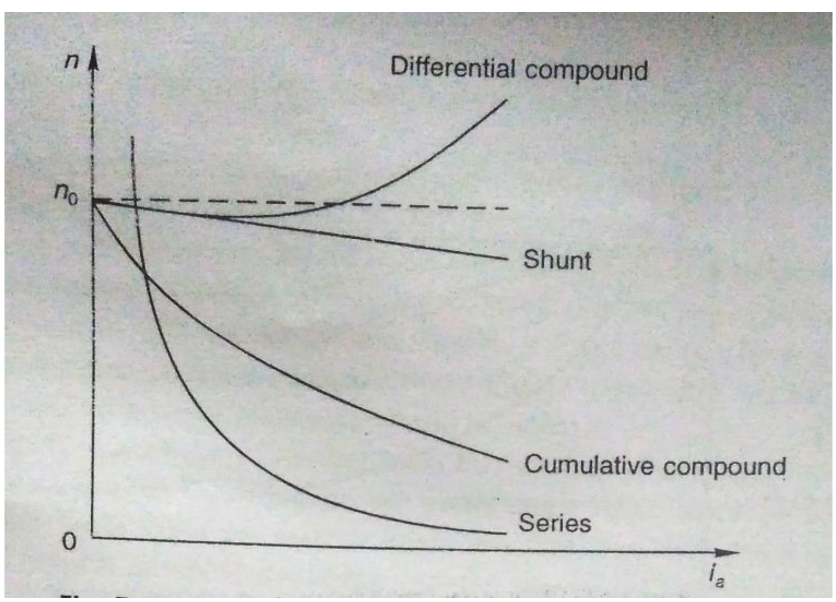




**Fig: Speed vs. Armature current in a DC Compound Motor Compared with other Motors**

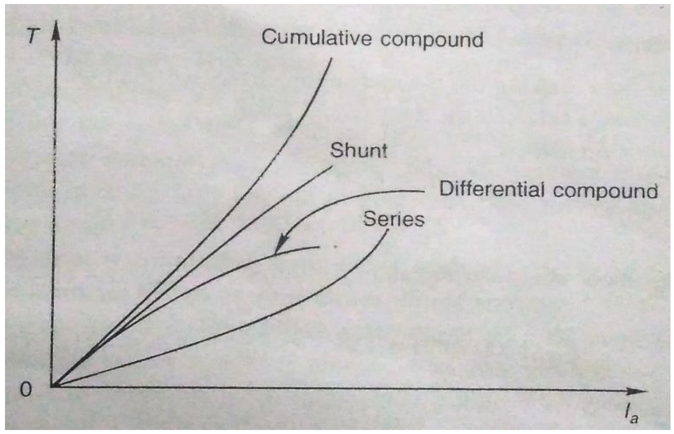



**Fig: Torque vs. Armature current in a DC Compound Motor Compared with other Motors**

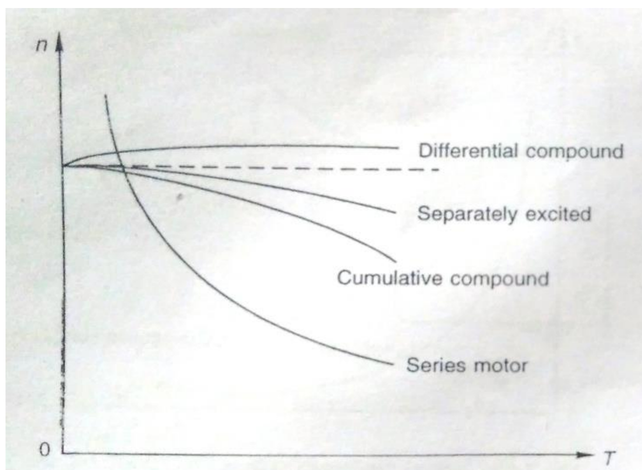



**Fig: Speed vs. Torque in a DC Compound Motor Compared with other Motors**

**The Torque-Speed Characteristic of a Cumulatively Compounded DC Motor :**

- In the cumulatively compounded DC motor, there is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load) which aid each other. Hence the flux per pole increases with armature current and as consequence (n-Ia) curve lies between that of a shunt motor( $\emptyset$ – constant ) and series motor ($\emptyset \propto Ia$ ).
- Therefore, the cumulatively compounded motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current).
- In a sense, the cumulatively compounded DC motor combines the best features of both the shunt and the series motors. Like a series motor, it has extra torque for starting; like a shunt motor, it does not over speed at no load.
- At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt DC motor. As the load gets very large, the series flux becomes quite important and the torque-speed curve begins to look like a series motor's characteristic. A comparison of the torque-speed characteristics of each of these types of machines is shown in Figures.

**The Torque-Speed Characteristic of a Differentially Compounded DC Motor:**

- In a differentially compounded dc motor, *the shunt magneto motive force and series magneto motive force subtract from each other.* This means that as the load on the motor increases, $\mathbf{I_a}$ increases and *the flux in the motor decreases.* But as the flux decreases, the speed of the motor increases. This speed increase causes another increase in load, which further increases $\mathbf{I_a}$ further decreasing the flux, and Increasing the speed again. The result is that a differentially compounded motor is unstable and tends to run away. This instability is *much* worse than that of a shunt motor with armature reaction. It is so bad that a differentially compounded motor is unsuitable for any application.
- Because of the stability problems of the differentially compounded DCmotor, it is almost never *intentionally* used.
- However, a differentially compounded motor can result if the direction of power flow reverses in a cumulatively compounded generator. For that reason, if cumulatively compounded DC generators are used to supply power to a system, they will have a reverse-power trip circuit to disconnect them from the line if the power flow reverses. No motor- generator set in which power is expected to flow in both directions can use a differentially compounded motor, and therefore it cannot use a cumulatively compounded generator.

Typical terminal characteristics of differentially compounded dc motor are also included in the Figures.

**Speed control of DC Motors:**

Speed control of DC Motors is easier as compared to the speed control of AC motors and much wider range of speeds is possible. That is one reason why even today they are preferred in modern industrial drives. From the two basic equations of DC machines

- $\mathbf{E_b = K_a. \Phi.\omega}$
- $\mathbf{V_T = E_b + I_a.R_a}$

We have the expression for the speed $\omega = (V_T - I_a.R_a) / K_a.\Phi$. From this equation we can (Since $K_a$ is a constant and $I_a$ is load dependent) easily see that the speed can be controlled by two methods:

1. By varying the terminal voltage known as : ***Armature Voltage Control ( AVC )*** and
2. By varying the field current and thus the flux per pole **Φ** known as **: *Flux control***

Let us study them one by one for all the three types of Motors

**Speed control of DC Shunt Motor:**

**Armature Voltage Control (AVC):**

This method involves changing the voltage applied to the armature of the motor without changing the Voltage applied to the field. This is possible with a separately excited DC Motor only and not with DC Shunt Motor. So first we shall explain for a DC separately excited motor and extend the same logic to a

shunt Motor. If the armature terminal Voltage $V_T$ is increased, then the $I_A$ will rise since **[ $I_A = (V_T \uparrow - E_b)/R_A$].** As $I_A$ increases, the induced torque **$\tau = K_a.\ \Phi.I_a\uparrow$ increases**, making **$\tau_{ind} > \tau_{load}$** , and the speed of the motor increases.
But, as the speed **ω** increases, **$E_b = K_a.\ \Phi.\omega\uparrow$** increases, causing the armature current $I_A$ to decrease since **[ $I_A = (V_T - E_b\uparrow)/R_A$].** This decrease in $I_A$ decreases the induced torque, causing **$\tau_{ind}$** to become equal to **$\tau_{load}$** at a final higher steady state rotational speed **ω**. Thus we can see that an increase in Armature voltage results in a higher speed and the resulting Speed Torque characteristics with **AVC** is shown in the figure below.

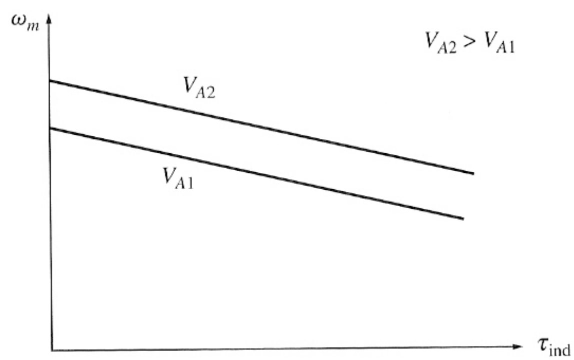



**Fig: The effect of armature voltage speed control**

Notice that the no-load speed of the motor is shifted by this method of speed control , but the slope of the curve remains constant
The cause-and-effect behavior in this method of speed Control can be summarized as below:

1. An increase in $V_T$ increases **[ $I_A = (V_T \uparrow - E_b)/R_A$**
2. Increasing $I_A$ increases **$\tau_{ind} = K_a.\ \Phi.I_a\uparrow$**
3. Increasing **$\tau_{ind}$** makes **$\tau_{ind}$** > **$\tau_{load}$** increasing **ω**.
4. Increasing **ω** increases **$E_b = K_a.\ \Phi.\omega\uparrow$**
5. Increasing $E_b$ decreases **$I_A = (V_T - E_b\uparrow)/R_A$**
6. Decreasing $I_A$ decreases **$\tau_{ind}$** until **$\tau_{ind}$** = **$\tau_{load}$** corresponding to a higher **ω**.

*In the case of a DC Shunt motor since changing the voltage applied to the armature of the motor without changing the Voltage applied to the field is not possible, a Variable resistance is introduced in series with the Armature which results in a reduction in the Armature current $I_A$. Effectively reduction of Armature current is equivalent to reduction in Armature voltage as seen in the above logic. Hence we get the same type of Speed control as shown in the figure above except that the characteristic with **$V_{A2}$** represents the nominal rated speed and that with **$V_{A1}$** represents with additional resistance introduced in series with the Armature. With this method, speed control is possible but speed can only be reduced from the rated or nominal speed. Even for a separately excited DC Motor it can provide speed control below Base speed only because armature voltage cannot exceed the rated value.*

**Inserting a resistor in series with the armature circuit** : If a resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque-speed characteristic, making it operate more slowly if loaded as shown in the figure below. This fact can easily be seen from the basic Equation:

$$\omega = (V_T / K_a . \Phi) -- [R_a / (K_a . \Phi)^2].\tau$$

The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.

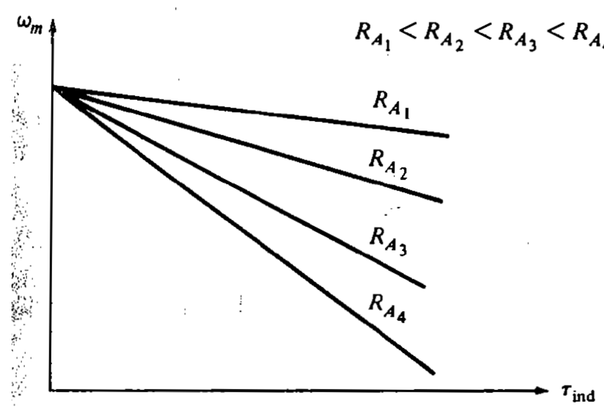
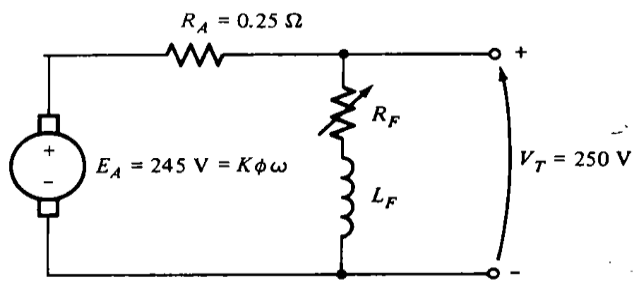



**Figure: Effect of Armature Resistance on the Speed Torque characteristic of a DC shunt motor**

**Flux control:**

Another method of Shunt motor speed control is to change the flux in the field. In a shunt motor Field current and hence field flux cannot be changed without changing the armature voltage. Hence flux control in Shunt motor is achieved by changing the Field resistance. Field coil resistance being fixed we cannot reduce it but increase the field circuit resistance by adding a variable resistance in series with the field coil as shown in the figure below.




Accordingly, when the resistance increases, the field current decreases **($I_F$↓ = $V_T/R_F$↑),** and as the field current decreases, the flux decreases. A decrease in flux causes an instantaneous decrease in the back emf ( **$E_b$↓ = $K_a$. Φ↓.ω )** which causes an increase in the machine's armature current since,

**$I_A$↑ = ($V_T$ -- $E_B$↓)/$R_A$**

The induced torque in a motor is given by **$\tau_{ind}$ = $K_a$. Φ↓.$I_a$↑ .**

*Here since the flux in this machine decreases while the current **$I_A$** increases, which way does the induced torque change?*

From practical data it is seen that for an increase in field resistance the decrease in flux is much lesser than the increase in armature current i.e. the increase in current predominates over the decrease in flux.

Hence, **$\tau_{ind}$** increases i.e. **$\tau_{ind}$** > **$\tau_{load}$** , and the motor speeds up.
However, as the motor speeds up, **$E_b$** rises, causing **$I_A$** to fall. Thus, induced torque **$\tau_{ind}$** too drops, and Finally **$\tau_{ind}$** equals **$\tau_{load}$** at a higher steady-state speed than the original speed .

The cause-and-effect behavior involved in this method of speed control is summarized below :

1. Increasing **$R_F$** causes **$I_F$↓ = $V_T/R_F$↑** to decrease
2. Decreasing **$I_F$↓** decreases **Φ↓**
3. Decreasing **Φ↓** lowers **$E_b$↓ = $K_a$. Φ↓.ω**
4. Decreasing **$E_b$↓** increases **$I_A$** since **$I_A$↑ = ($V_T$ -- $E_B$↓)/$R_A$**
5. Increasing **$I_A$** increases **$\tau_{ind}$ = $K_a$. Φ↓.$I_a$↑**, with the change in **$I_A$** being dominant over the change in flux ).
6. Increasing **$\tau_{ind}$** makes **$\tau_{ind}$** > **$\tau_{load}$**, and the speed **ω** increases.
7. Increasing **ω** increases **$E_b$↑= $K_a$. Φ.ω↑** again.
8. Increasing **$E_b$** decreases **$I_a$**
9. Decreasing **$I_a$** decreases " **$\tau_{ind}$** until " **$\tau_{ind}$** = " **$\tau_{load}$** at a higher speed **ω**.

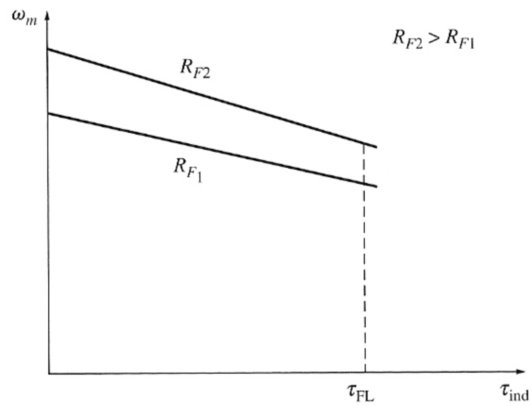



**Fig: Shunt Motor Speed control with Flux control (Change in field resistance)( over the normal operating Range )**

The Speed Torque characteristics with change in Field Resistance are shown in the figure below. Notice that with flux control i.e. with insertion of additional resistance in the field circuit, the flux in the machine decreases and hence:

- The no- load speed of the motor increases, while the slope of the torque-speed curve becomes steeper and also
- Speeds above base speed only can be achieved. (as against with Armature resistance insertion control , speeds below base speed only can be achieved) since to achieve speed below base speeds field current has to be increased beyond its rated value which is not permitted. In a normally designed motor the maximum speed can be twice the rated speed and in specially designed motors it can be up to six times the rated speed.

**Other important Limitation of field resistance speed control**:

The effect of increasing the field resistance on the output characteristic of a DC shunt motor as seen and explained above is a consequence of the Equation

$$\omega = (V_T / K_a.\ \Phi) \text{ -- } [R_a / (K_a.\ \Phi)^2].\tau$$

which describes the technical characteristic of the motor. In this equation, the no-load speed is proportional to the reciprocal of the flux in the motor, while the slope of the curve is proportional to the reciprocal of the flux squared. Therefore, a decrease in flux causes the slope of the torque- speed curve to become steeper. The earlier figure shows the technical characteristic of the motor over the range from no-load to full-load conditions. Over this range, an increase in field resistance increases the motor's speed, as described above. Hence for motors operating between no- load and full-load conditions, an increase in $\mathbf{R_F}$ may reliably be expected to increase the operating speed.

Now let us examine the figure shown below. This figure shows the technical characteristic of the motor over the full range i.e. from no- load to stall conditions. It is apparent from the figure that at *very slow* speeds an increase in field resistance will actually *decrease* the speed of the motor. This effect occurs because , at very low speeds, the increase in armature current caused by the decrease in $\mathbf{E_b}$. is no longer large enough to compensate for the decrease in flux in the induced torque equation. With the flux decrease being actually larger than the armature current increase, the induced torque decreases, and the motor slows down.

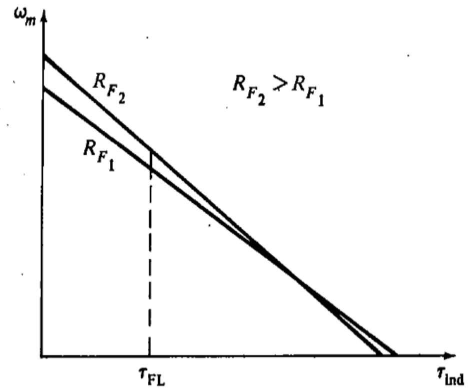



**Fig: Shunt Motor Speed control with Flux control (Change in field resistance)( over the complete operating Range i.e. from no load to stall condition)**

Some small DC motors used for control purposes actually operate at speeds close to stall conditions. For these motors, an increase in field resistance might have no effect, or it might even decrease the speed of the motor. Since the results are not predictable, field resistance speed control should not be used in these types of dc motors. Instead, the armature voltage method of speed control should be employed.

**Speed Control of Series DC Motors:**

Unlike with the shunt dc motor, there is only one efficient way to change the speed of a series dc motor. That method is to change the terminal voltage of the motor. If the terminal voltage is increased, the first term in Equation

$$\omega = [V_T / \sqrt{(K_{af}.\tau)}] -- [R_a/(K_{af})]$$

is increased, resulting in a *higher speed for any given torque.*

The speed of DC series motors can also be controlled by the insertion of a series or parallel (Diverter) resistor into the motor circuit as shown in the figures below along with the resulting effect on Speed torque characteristics. But in this technique large amount of power is dissipated as heat and thus wasted. Hence this method is used only for intermittent periods during the start-up of some motors.

Until the last 40 years or so, there was no convenient way to change $\mathbf{V_T}$, so the only method of speed control available was the wasteful series resistance method. That has all changed today with the introduction of solid-state control circuits. We will study the techniques of obtaining variable terminal voltages subsequently in another subject '***Power Electronics*** '

**Speed Control of Cumulatively Compounded DC Motor:**

The techniques available for the control of speed in a cumulatively compounded DC motor are the same as those available for a shunt motor:

1. Change the field resistance $\mathbf{R_F}$
2. Change the armature voltage $\mathbf{V_T}$
3. Change the armature resistance $\mathbf{R_A}$.

The analysis describing the methods and effects of changing $\mathbf{R_F}$ or $\mathbf{V_T}$ or $R_A$ are similar to the analysis given earlier for the shunt motor.

**The ward Leonard system:**

The speed of a separately excited, shunt, or compounded dc motor can be varied in one of three ways: by changing the field resistance, changing the armature voltage, or changing the armature resistance. Of these methods, perhaps the most useful is armature voltage control, since it permits wide speed variations without affecting the motor's maximum torque.
A number of motor-control systems have been developed over the years to take advantage of the high torques and variable speeds available from the armature voltage control of DC motors. In the days before solid-state electronic components became available, it was difficult to produce a varying DC volt age. In fact, the normal way to vary the armature voltage of a dc motor was to provide it with its own separate dc generator.

An armature voltage control system of this type known as ward Leonard Speed control system is shown in the Figure below .

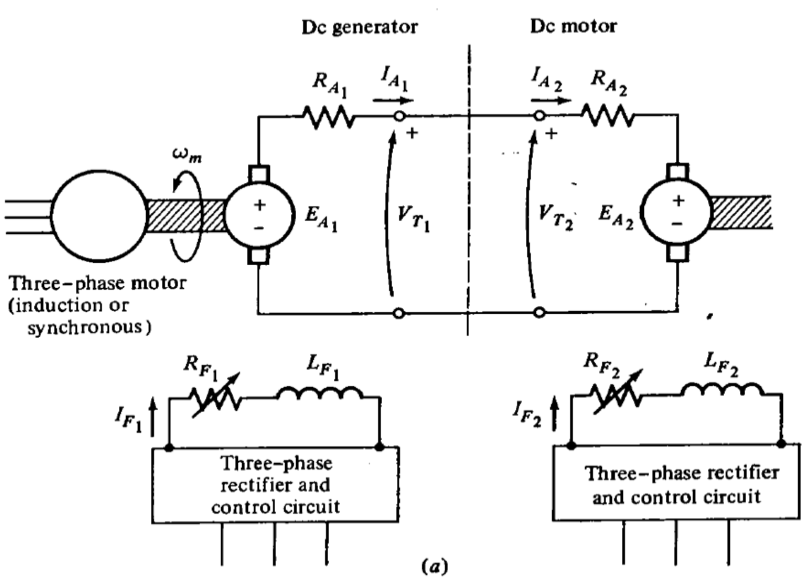



**Figure: Ward Leonard DC Motor Speed control system**

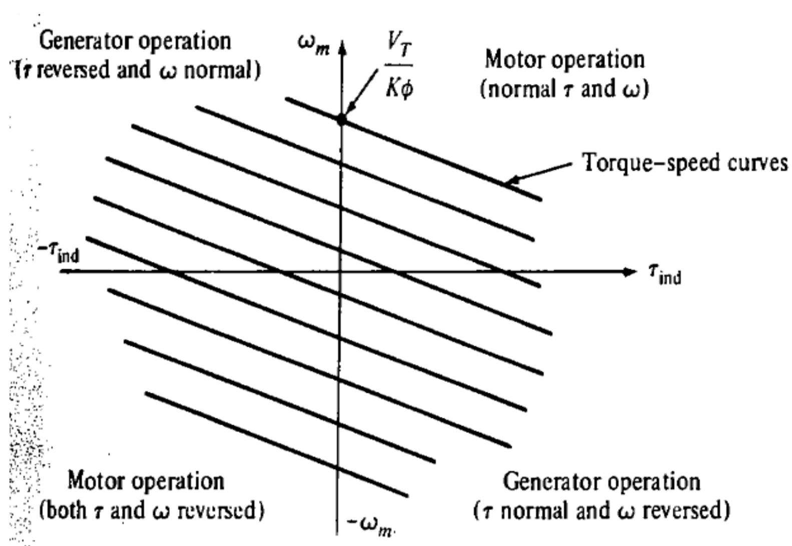



**Figure: The operating range of a Ward-Leonard motor-control system. The motor can operate as a motor in either the forward (quadrant - 1) or reverse (quadrant -3) direction and it can also regenerate in quadrants 2 and 4.**

In this an AC motor is serving as a prime mover for a DC generator, which allows the motor's speed to be smoothly varied between a very small value and the base speed. The speed of the motor can be adjusted above the base speed by reducing the motor's field current. With such a flexible arrangement, total motor speed control is possible.

Furthermore, if the field current of the generator is reversed, then the polarity of the generator's armature voltage will be reversed, too. This will reverse the motor's direction of rotation. Therefore, it is possible to get a very wide range of speed variations in *either direction of rotation* using a Ward-Leonard DC motor control system.

Another advantage of the Ward-Leonard system is that it can "regenerate," or return the machine's energy of motion to the supply lines. If a heavy load is first raised and then lowered by the DC motor of a Ward-Leonard system, when the load is being lowered, the DC motor acts as a generator and supplying power back to the power system. In this fashion, much of the energy required to lift the load in the first place can be recovered, reducing the machine's overall operating costs.

The possible modes of operation of the DC machine are shown in the torque- speed diagram shown in the above Figure. When this motor is rotating in its normal direction and supplying a torque in the direction of rotation, it is operating in the first quadrant of this figure. If the generator's field current is reversed, that will reverse the terminal voltage of the generator, in turn reversing the motor's armature voltage. When the armature voltage reverses with the motor field current remaining unchanged, both the torque and the speed of the motor are reversed, and the machine is operating as a motor in the third quadrant of the diagram. If the torque or the speed alone of the motor reverses while the other quantity does not, then the machine serves as a generator, returning power to the dc power system. Because a Ward-Leonard system permits rotation and regeneration in either direction, it is called *a four-quadrant control system.*

The disadvantages of a Ward-Leonard system should be obvious. One is that the user is forced to buy *three* full machines of essentially equal ratings, which is quite expensive. Another is that three machines will be much less efficient than one. Because of its expense and relatively low efficiency, the Ward-Leonard system has been replaced in new applications by SCR-based controller circuits.

## Principle of 3 point and 4 point starters:

Before studying the principle of operation of these starters let us understand the basic principles underlying the starters.

- DC motors are by themselves self starting type. Once the appropriate field and armature supply are given the motors start automatically. They do not need any additional device for the purpose of starting.
- But DC motor starters are required for safe starting of the motors. Initially just at the starting of the motor, the speed is zero and hence the back emf $\mathbf{E_b}$ is also zero. In this condition if the Rated terminal voltage $\mathbf{V_t}$ is applied to the motor we can see from the basic governing equation

$$\mathbf{I_a = (V_t - E_b)/R_a}$$

  That the motor draws excessive current which would be easily 10 to 15 times that of the nominal rated current of the motor. This excessive current would flow till the motor develops the rated speed.
- During this transient period when the excessive current flows the torque developed also would be excessive. With the result the motor would get damaged both electrically and mechanically.

- To protect the motor from such damage, a resistance is introduced in series with the motor as shown in the figure below which would be withdrawn gradually in steps as the motor picks up speed.
- This is a basic arrangement of a DC motor starter and its operation is totally manual. But practical starters have been developed with additional protective and automatic starting features. They are called 3 ***point starters and 4 point starters***, the subject of our study.

## 3 Point starter:

The circuit diagram and the arrangement of a three point starter are shown in the figure below.

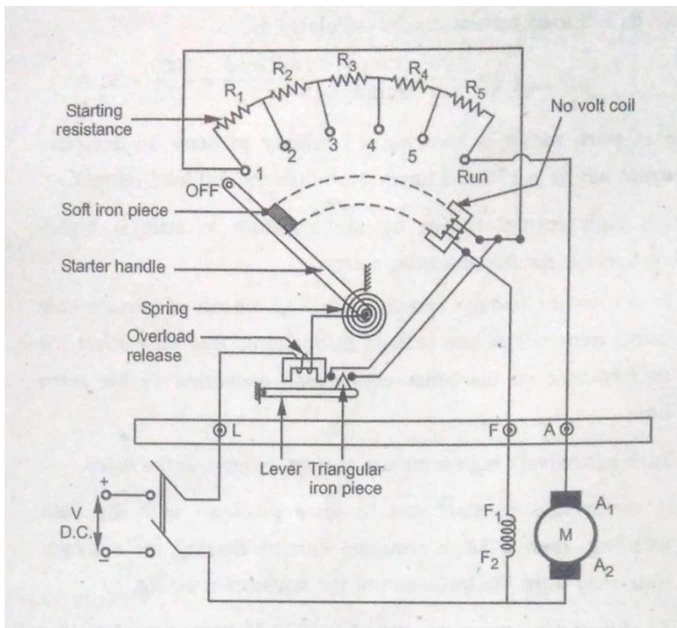



**Figure: 3 Point starter**

**Basic features and working principles:**

- The basic component viz starter resistance comes in steps with contact points brought out as studs 1,2,3.. Run.
- The three points are:
  - L – The line terminal to be connected to the DC positive terminal through a two pole switch
  - A – The terminal to be connected to the terminal $A_1$ of the armature.
  - F – The terminal to be connected to the terminal $F_1$ of the Field winding
- The other ends $A_2$ of the armature and $F_2$ of the field are connected to the other contact of the two pole switch which gets connected to the negative terminal of the DC power supply when switched on.

- Point L in turn is connected to the pivot point of the handle through a protective device called OLR ( Over Load Relay )
- The handle which is spring loaded comes back to the OFF position under its own force until locked in the RUN position due to the electromagnetic pull of the other protective device known as NVC ( No Volt Coil)
- The field terminal F is connected to starting point 1 of the resistance in a parallel path through the NVC.

**Operation of the starter:** The starter is gradually moved from the initial position to the final RUN position manually against the spring force. When the handle comes in contact with stud -1 , the field supply gets extended to the field coil through the parallel path connected directly from stud -1 through NVC . In the starter initially entire resistance comes in series with the armature and as the handle is moved towards RUN, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Finally when the handle is brought to the Run position, the entire resistance gets removed from the Armature circuit and the motor runs at the rated speed. The handle is held in RUN position due to the action of the NVC.

**Action of the NVC:** When the field current flows through the NVC it attracts the handle with the soft iron piece and keeps it in contact the NVC electromagnet. Hence NVC is also called as ***Hold On Coil***. In addition to holding the handle in the final RUN position, the NVC works as a safety/protection device by releasing the handle back to the start position from the RUN position whenever there is a power failure or when the field circuit breaks. Thus the entire starting resistance comes into the armature circuit every time the motor is started from zero speed and prevents high inrush currents during every fresh starting attempt after a power failure.

**Action of the OLR:** As can be seen from the figure there is another protective relay called OLR (Over Load Relay) which is also an electromagnet which works in conjunction with an arm fixed on a fulcrum at one end and with a triangular iron piece fixed on the other end. Whenever there is an overload current beyond a set safety value, the electromagnet activates and pulls the arm upwards and the triangular iron piece short circuits the two terminals which are connected to the two ends of the NVC coil. Thus with any overload due to a fault in the motor or associated circuit , the NVC gets deactivated and releases the handle back to the initial safe start condition. After the fault is rectified the motor can be started afresh with full resistance brought back into the armature circuit.

### 3 Point starter with a brass/copper arc:

In the earlier version of the 3 Point starter as we have seen, as the handle is moved towards RUN position, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Thus finally when the motor is running, the entire starter resistance gets added to the field circuit. But since the starting resistance value is very small compared to field winding resistance, this hardly reduces the field current and hence there is no any practical impact. However this addition of resistance in the field circuit can be avoided by providing a brass or copper arc with one end connected to the stud -1 and the other end connected to the NVC as shown in the figure below.

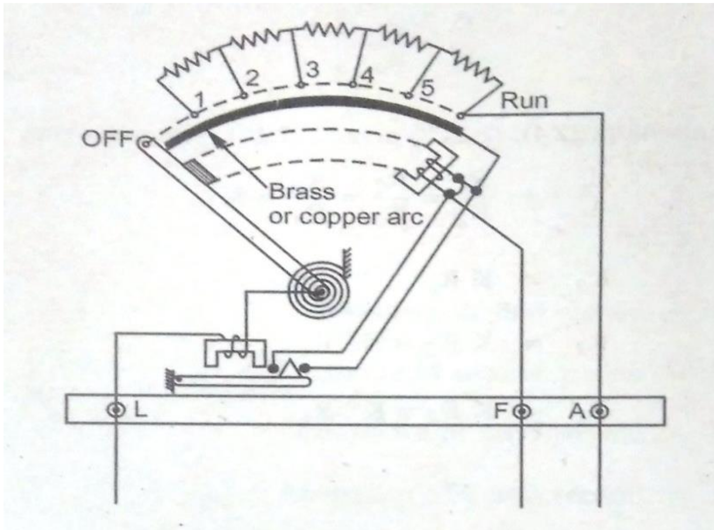



**Figure: 3 Point starter with Brass arc**

With such an arrangement when the handle moves on the arc the field current directly flows through the arc to the NVC thus avoiding the starting resistance. With such an arc in place, the earlier parallel connection from stud -1 to the NVC start terminal is no more required and hence is removed.

**4 Point starter:** The operation of the 4 point starter is explained along with the schematic diagram shown below.

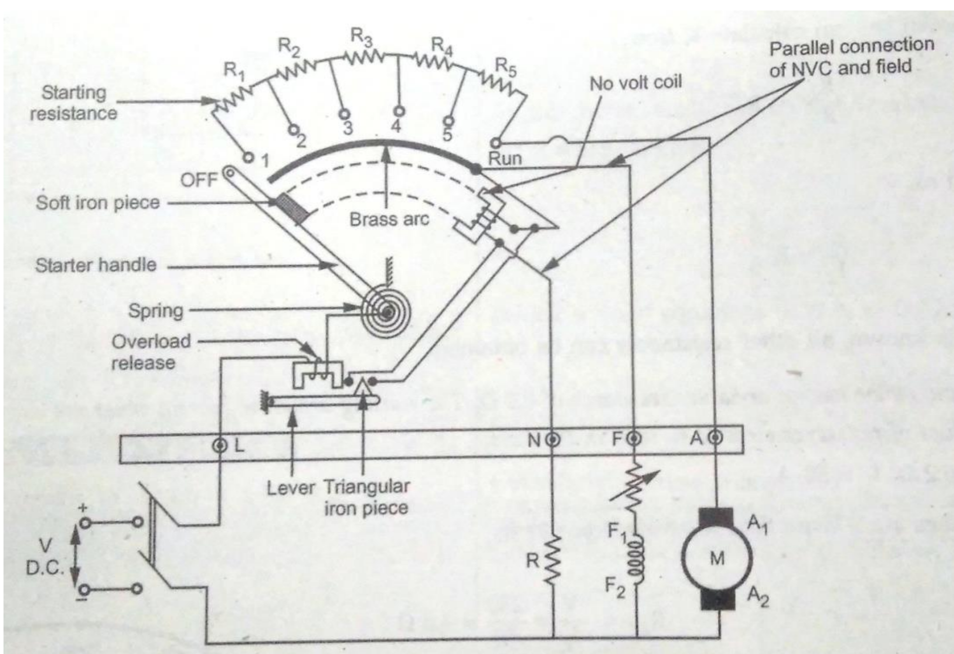



**Figure: 4 Point starter**

- The basic difference between a 3 Point starter and a four point starter is : In a 3 point starter NVC was connected in series with the field coil while in a 4 point starter the NVC is connected independently to the supply through a fourth terminal termed as N in addition to L, F and A.
- With this arrangement any change in the field current due to the change field control resistance will not affect the performance of the NVC. This ensures that NVC always produces a force enough to hold on the handle irrespective of the amount of field current. Adequate current required for the confirmed operation of the NVC is obtained by adjusting the resistor R connected in series with the NVC coil.
- However the 4 point starter has a separate disadvantage: Since now the NVC is connected separately excluding the field current, it cannot detect the field failure and hence the resulting ***over speed*** cannot be prevented.

# Important concepts and Formulae:

- **Torque generated in a DC machine :** $\tau = K_a . \Phi . I_a$
- **KVL around the armature circuit:** $V_T = E_b + I_a . R_a$
- **Generalized *Torque vs. Speed* equation in different types of motors:**

$$\omega = (V_T / K_a . \Phi) - [R_a / (K_a . \Phi)^2] . \tau$$

- **Shunt Motor's Other Characteristics:**

  **Speed vs. $I_a$ :** $\omega = (V_T - I_a . R_a) / K_a . \Phi$

  **Torque vs. $I_a$:** $\tau = K_a . \Phi . I_a$
- **Series Motor:**

$$\omega = [V_T / \sqrt{(K_{af} . \tau)}] - [R_a / (K_{af})]$$

- **Speed vs. $I_a$ :** $\omega = (V_T - I_a . R_a) / K_{af} . I_a = (V_T / K_{af} . I_a) - (R_a / K_{af})$
- **Torque vs. $I_a$:** $\tau = K_a . \Phi . I_a = K_{af} . I_a^2$
- **Speed control with armature voltage control is possible only below the rated or nominal speed (also known as base speed).**
- **Speed control with flux control is possible only above the base speed**

# Illustrative examples:

**Ex.1:** A 500 V shunt motor with $R_f = 250\ \Omega$ and $R_a = 0.2\ \Omega$ runs at 2500 RPM taking a current of 25 A from the mains supply . Calculate the resistance to be added to the armature circuit to reduce the speed to 1500 RPM keeping the armature current constant.

**Solution:**

***First let us calculate the back e.m.f developed by the motor in the given first set of conditions:***

Field current $I_f$ = Rated terminal voltage / $R_f$ = 500 / 250 = 2 A

Armature current $I_a = I_l - I_f = (25 - 2) = 23$ A

Back e.m.f $E_b = V_T - I_a R_a = 500 - 23 \times 0.2 = 495.4$ V

We know that the back e.m.f is proportional to the speed

$\therefore$ $E_{b1} / E_{b2} = N_1 / N_2$ i.e 495.4 / $E_{b2}$ = 2500/1500 $\therefore$ $E_{b2}$ = 495.4 x 1500 / 2500

= 297.24 V

But we also know that $E_{b2} = V_T - I_a R_{a2}$ ( Since the terminal voltage and the armature current remain the same )

$\therefore$ 297.24 = 500 – 23 x $R_{a2}$ from which we get $R_{a2}$ = ( 500 – 297.24 ) /23 = 8.82 Ω

This is the total new resistance of the armature circuit (including the original armature resistance of 0.2 Ω to get a speed of 1500 RPM)

**Hence the new resistance to be added into the armature circuit** = 8.82- 0.2 = **8.62 Ω**

**Ex.2:** A DC shunt motor takes 22 A from 250 V supply. $R_a$ = 0.5 Ω , $R_f$ = 125 Ω. Calculate the resistance required to be connected in series with the armature to halve the speed (a ) when the load torque is constant ( b) When the load torque is proportional to the square of the speed

**Solution :**

*First let us calculate the speed of the motor when the load current $I_l$ is 22 A :*

Field current $I_f$ = Rated Terminal voltage / Field resistance = 250/125 = 2 A

Armature current $I_a = I_l - I_f$ = 22 -2 = 20 A

Back e.m.f $E_b = V_T - I_a R_a = 250 - 20 \times 0.5 = 240$ V

(a) *we have to find out the New $R_a$ when the speed is halved with torque maintained constant* :

We know that Torque $T = K_a . Ø . I_a$ . In this case since change is only in the armature resistance field current and hence flux Ø remains the same. Further since the torque is maintained constant the armature currents are also equal and hence $I_{a1} = I_{a2}$ = 20 A

We also know that $E_b = K_a . Ø . \omega$ . As already explained, $K_a$. Ø remains same and hence when the speed is halved the back e.m.f also gets halved.

Hence $E_{b2}$ = 120 V = $V_T - I_a R_{a2}$ i.e 250 – 20 x $R_{a2}$ = 120 V i.e $R_{a2}$ = (250 -120)/20 = 6.5Ω

Hence the **Resistance to be added to halve the speed** = $R_{a2} - R_a$ = 6.5-0.5 = **6.0 Ω**

**(b)** *Next we have to find out the New $R_a$ when the speed is halved when torque is proportional to square of speed.*

When the torque is proportional to the square of the speed $\tau_1 = K\,\omega_1^2$ and $\tau_2 = K\,\omega_2^2$

$\therefore \quad \tau_1 / \tau_2 = K\,\omega_1^2 / K\,\omega_2^2 = \omega_1^2 / \omega_2^2 = (1/0.5)^2 = 4$

But Torque is also proportional to the product of flux (and hence field current )and Armature current. Here field circuit is not disturbed and hence the field current is same. Using this relation we can find out new armature current $I_{a2}$

$\therefore \quad \tau_1 / \tau_2 = K \times I_f \times I_{a1} / K \times I_f \times I_{a2} = I_{a1} / I_{a2} = 4$ i.e $I_{a2} = I_{a1} / 4 = 20/4 = 5$ A

Next using the relation between the speeds and the back emfs we can find out the armature resistance to be added.

$\omega_1 / \omega_2 = 2$ and also

$\omega_1 / \omega_2 = E_{b1}/E_{b2} = 240 / (250 - 5 \times R_{a2})$ i.e $250 - 5R_{a2} = 240/2 = 120$ From which we get

$R_{a2} = (250-120)/5 = 26\,\Omega$ $\therefore$ Finally **Resistance to b added** is = 26-0.5 = **25.5 Ω**

**Ex.3:** A 250 V DC series motor takes 40 A and runs at 1000 RPM. Find the speed at which it runs if its torque is halved. Assume that the motor is operating in the unsaturated region of its magnetization. $R_f = 0.25\,\Omega$ $R_a = 0.25\,\Omega$

*First we will use the relation between torque and armature current and get the back e.mf when the torque is halved* :

In a DC motor we know that the torque is proportional to $\emptyset . I_a$. In the case of a series DC motor flux is proportional to the armature current itself since $I_f = I_a$ . Hence in a series motor $\boldsymbol{\tau \propto I_a^2}$

Hence $\boldsymbol{\tau_1/\tau_2 = I_{a1}^2 / I_{a2}^2} = 2$ (Since torque is halved) $\boldsymbol{I_{a1} / I_{a2}} = \sqrt{2}$

$I_{a1} = 40$ A and $I_{a2} = 40/\sqrt{2} = 28.28$ A

$E_{b1} = 250 - 40\,(0.25 + 0.25) = 230$ V and $E_{b2} = 250 - 28.28\,(0.25 + 0.25) = 235.86$ V

*Next we will use the relation between back emf and speed and get the speed when the torque is halved:*

We know that $E_{b1} = K_a\,\emptyset_1 N_1$ and $E_{b2} = K_a\,\emptyset_2 N_2$ . But since the flux is proportional to $I_a$ the relations become $E_{b1} = K\,I_{a1}N_1$ and $E_{b2} = K\,I_{a2}N_2$ where K is a new constant.

Hence $E_{b1}/E_{b2} = K\,I_{a1}N_1 / K\,I_{a2}N_2 = I_{a1}N_1 / I_{a2}N_2$ and $N_2 = (I_{a1}/I_{a2})\,(E_{b2}/E_{b1})\,N_1$

Substituting the above values we get $\boldsymbol{N_2} = \sqrt{2}\,(235.86/230)1000 =$ **1450 RPM**

**Ex.4:** A 500 V DC shunt motor runs at 1900 RPM taking an armature current of 150 A. The armature resistance is 0.16 Ω. Find the speed of the motor when a resistance is inserted in the field circuit which reduces the field current to 80 % and the armature current is 75 A.

**Solution:**

We know that the back e.m.f of a DC motor is proportional to the Flux and speed. And in the unsaturated region of the magnetization region the flux in turn is proportional to the field current. So Back e.m.f is proportional to field current and speed. We will find out the new speed by calculating the back e.m.fs [from the relation ($E_b = V_T - I_a R_a$)]and using the above proportionality relation in both the conditions as below.

$E_{b1} = V_T - I_{a1} R_a = 500 – 150 \times 0.16 = 476$ V and is equal to $K_a.Ø_1. N_1$

$E_{b2} = V_T - I_{a2} R_a = 500 – 75 \times 0.16 = 488$ V and is equal to $K_a . 0.8Ø_1. N_2$

$\therefore$ $476 /488 = K_a.Ø_1. N_1 / K_a . 0.8Ø_1. N_2$

And $\mathbf{N_2} = (488 /476)( N_1 / 0.8 ) = (488 /476)( 1900 / 0.8 ) =$ **2435 RPM**

# UNIT – V

# Testing of D.C. Machines

- Losses – Constant & Variable losses
- Calculation of efficiency
- Condition for maximum efficiency.
- Methods of Testing
    - Direct, indirect and regenerative testing
    - Brake test
    - Swinburne's test
    - Hopkinson's test
    - Field's test
    - Retardation test
- Separation of stray losses in a DC motor test
    - **Important concepts and Formulae**
    - **Illustrative examples**

## Losses:

DC Generators convert Mechanical power into Electrical power and DC Motors convert Electric power to Mechanical power. In the process of conversion some power is lost. The difference between the input power and the output power of a machine is the **Power loss** that occurs inside the machine.

## Constant & Variable losses:

The losses are broadly classified as *constant losses* and *variable losses*. Constant losses are constant and are independent of the load where as the variable losses are dependent on the load. They are further classified in detail as below.

**Detailed Classification of Losses:**

1. **Electrical or Copper Losses ($I^2R$ Loss):** Current flow through the resistance of Armature and Field coils gives rise to $I^2R$ losses and since the coils are normally made up of copper these losses are called Copper losses.
   Armature copper loss: $P_A = I_A^2 R_A$
   Field copper loss: $P_F = I_F^2 R_F$

2. **Brush losses**: The brush drop loss is the power lost across the contact potential at the brushes of the machine. It is given by the equation:

$$\mathbf{P_{BD} = V_{BD} \times I_A}$$

   where $\mathbf{P_{BD}}$ = brush drop loss
   $\mathbf{V_{BD}}$ = brush voltage drop
   $\mathbf{I_A}$ = armature current

   The brush losses are calculated in this manner because the voltage drops across a set of brushes are approximately constant over a large range of armature currents. Unless otherwise specified. The brush voltage drop is usually assumed to be about 2 V.

3. **Core Losses:** *Hysteresis* and *eddy current* losses occurring in the Armature and Field cores together are called core losses.
   - **Hysteresis loss**: in an iron core is the loss of power due to the hysteresis loop in the magnetization characteristic of the core in each cycle of the alternating current applied to the core. In the case of DC machines though there is no alternating current applied to the core, the change in the magnetic flux within the machine due to its constructional features result in a small *hysteresis loss*
   - **Eddy current losses**: A time-changing flux induces voltage within a ferromagnetic core in just the same manner as it induces voltage in the conductors around the core of the armature. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name ***eddy currents***. These eddy

currents flowing in a resistive material (the iron of the core) cause power loss thus heating the iron core and the resulting loss is called ***eddy current loss***. This loss is proportional to the thickness of the core material and hence to minimize this loss the core is made up of thin sheets called laminations instead of a single thick block. An insulating oxide or resin is used between the strips so that the current paths for eddy currents are limited to very small areas. Thus the eddy current losses have a very little effect on the core's magnetic properties.

4. **Mechanical Losses:** They are associated with the mechanical effects and they are mainly *Friction* and *windage* losses.
   - ***Friction losses*** are losses caused by the friction in the bearings of the machine and
   - ***Windage losses*** are due to the friction between the moving parts of the machine and the air flow in the machine housing.
5. **Stray Losses:** They are other miscellaneous losses that cannot be grouped into any of the above categories.

*Out of the above, Core Losses and Mechanical Losses are grouped under Constant losses. Electrical or Copper Losses and Stray Losses are grouped under variable losses.*

The losses and their classification explained above is summarized in the form of a tree and is shown below.

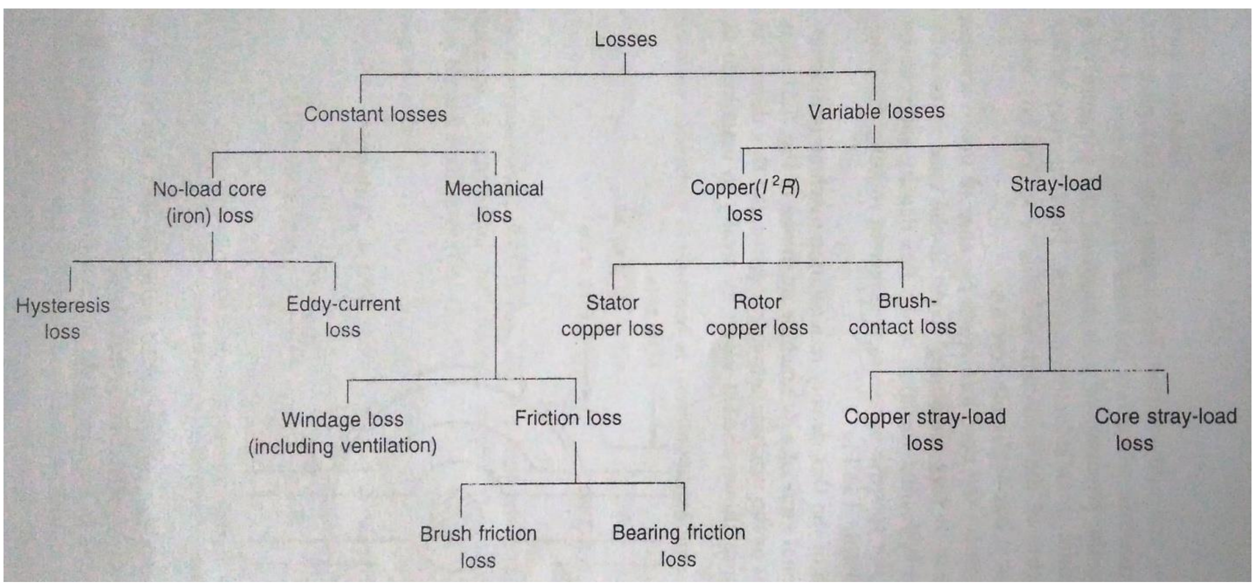



**Figure: Classification of losses in DC Machine**

The power flow in DC machines showing the stages where the different losses occur is shown clearly in the figure below.

**Power flow diagram:**

One of the most convenient techniques for accounting for power losses and showing them clearly in the order in which they occur in a machine is the *power-flow diagram.* A power-flow diagram for a DC generator is shown in the figure (a) below. In this figure, mechanical power is input into the machine, and then the stray losses, mechanical losses, and core losses are subtracted. After they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled $\mathbf{P_{conv}}$. The mechanical power that is converted is given by:

$$P_{CONV} = \tau_{ind} \cdot \omega_m$$

and the resulting electric power produced is given by: $\mathbf{P_{CONV} = E_A . I_A}$

However, this is not the power that appears at the machine's terminals. Before the terminals are reached, the electrical power losses like the copper losses and the brush losses must be subtracted.

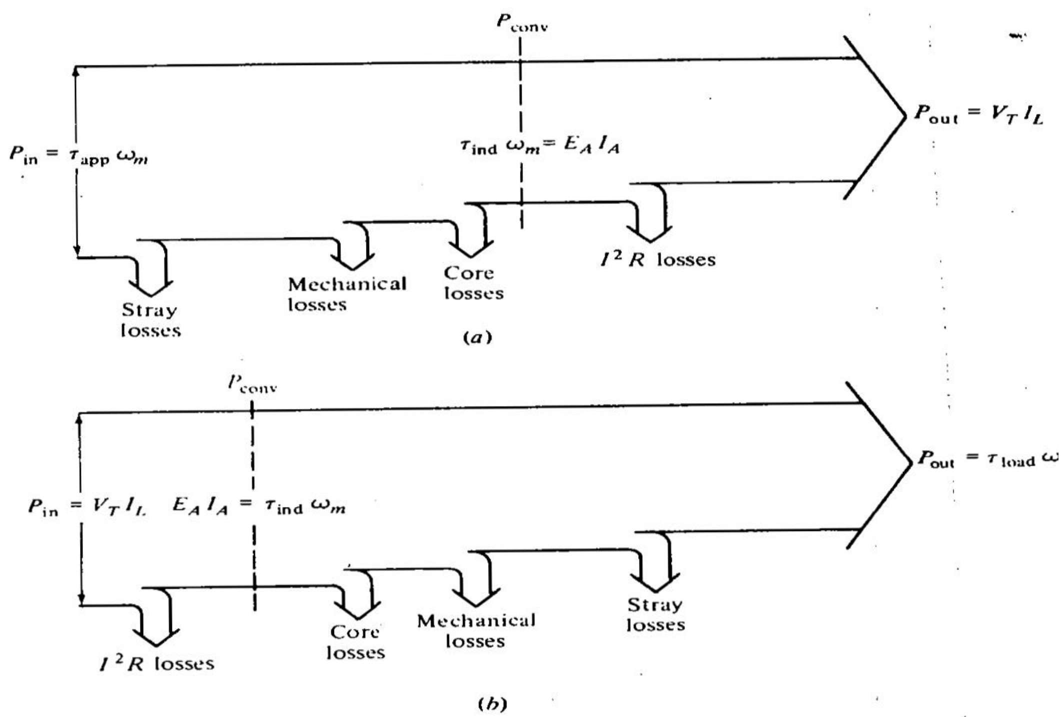



**Figure: Power flow diagram of a DC Machine (a) Generator (b) Motor**

In the case of dc motors, this power-flow diagram is simply reversed. The Power flow diagram for a motor is shown in the above figure (b) above

## Efficiency:

The efficiency of a DC Machine is defined as $\eta = (P_{out}/P_{in}).\ 100\ \% = [(P_{in} - P_{loss}) / P_{in}] \times 100\ \%$

*Using this basic relation and from a clear understanding of the above Power flow the* **η** *calculations when the machine is working as a Generator and as a Motor are given below.*

**Efficiency calculations of Generator:**

- If $I_L$ is the load current supplied by the Generator at a terminal voltage of $V_T$ then the output power is given by $P_{out} = V_T.I_L$
- The armature current $I_A = I_L + I_F$
- Armature copper loss $P_A = I_A^2 R_A$
- Field copper loss $P_F = I_F^2 R_F$
- Total losses $= I_A^2 R_A + I_F^2 R_F + W_C$ where $W_C$ is the sum of the core losses and stray losses. (also known as constant losses)
- Therefore Input $= P_{out} +$ **Total losses** $= P_{out} + I_A^2 R_A + I_F^2 R_F + W_C$

Hence $\eta = (P_{out}/P_{in}).\ 100\ \% = (V_T.I_L)/(V_T.I_L + I_A^2 R_A + I_F^2 R_F + W_C).100\%$

**Efficiency calculations of Motor:**

- If $I_L$ is the line current taken by the Motor at a terminal voltage of $V_T$ then the input power is given by $P_{in} = V_T.I_L$
- The losses are same as in the Generator
- Therefore output $P_{out} = P_{in} --$ **Total losses** $= P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)$

Hence $\eta = (P_{out}/P_{in}).\ 100\ \% = [\{P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)\}/ (V_T.I_L)]\ .\ 100\%$

Of these losses **( $I_F^2 R_F + W_C$) are** called constant losses **$P_C$** since they are almost independent of load. The armature copper losses i.e. **($I_A^2.R_A$)** is called the variable loss and is dependent on the load. The variable loss varies approximately as the square of load current. We say approximately since loss varies as the square of the armature current and not as the square of the load current. Hence if we know the loss at full load, the loss at half load, one fourth load etc can be calculated.

## Condition for Maximum efficiency:

The condition for maximum efficiency is developed by differentiating the expression for efficiency as a function of load current and equating it to zero since the variable losses are dependent on the load current.

***Generator:***

The efficiency is obtained as: **$\eta = (P_{out}/P_{in}).\ 100\ \% = (V_T.I_L)/(V_T.I_L + I_A^2 R_A + I_F^2 R_F + W_C).100\%$**

Neglecting the field current which is small compared to armature current we get

$$\eta = (V_T.I_L)/(V_T.I_L + I_L^2 R_A + W_C).100\% = 1/[1 + I_L^2 R_A/(V_T.I_L) + W_C/(V_T.I_L)].100$$
$$= 1/[1 + I_L R_A / V_T + (W_C/(V_T I_L)].100$$

The efficiency is maximum when the denominator is maximum. Hence the condition for maximum efficiency becomes: **$d/dI_L\ [1 + I_L R_A / V_T + (W_C/(V_T I_L)] = 0$ i.e. $R_A / V_T - (W_C/(V_T I_L^2) = 0$**

And finally the condition for maximum efficiency becomes: **$I_L^2 R_A = W_C$**

Which means **Variable losses = Constant Losses**

And the current at maximum efficiency becomes:

$$I_L = \sqrt{W_C/R_A} = \sqrt{\text{Constant Losses/Armature resistance}}$$

***Motor:***

The efficiency is obtained as: **$\eta = (P_{out}/P_{in}).\ 100\ \% = [\{P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)\}/(V_T.I_L)]\ .\ 100\%$**

$$= [\{V_T.I_L - (I_A^2 R_A + I_F^2 R_F + W_C)\}/(V_T.I_L)]\ .\ 100\%$$

Neglecting the field current which is small compared to armature current we get

$$\eta = [\{V_T.I_L - (I_L^2 R_A + W_C)\}/(V_T.I_L)]\ .\ 100\%$$
$$= 1 - [(I_L^2 R_A + W_C)/(V_T.I_L)]\ .\ 100$$

**$\eta$** becomes maximum when the term in the square brackets becomes minimum and thus the condition for maximum efficiency becomes **$d/d\,I_L\ [(I_L^2 R_A + W_C)/(V_T.I_L)] = 0$** which again finally becomes **:**

$$I_L^2 R_A = W_C$$

**Or Variable losses = Constant Losses**

And the current at maximum efficiency also becomes:

$$I_L = \sqrt{W_C/R_A} = \sqrt{\text{Constant Losses/Armature resistance}}$$

**Both same as that for the generator.**

## Testing of DC machines:

Involves the measurement of the various losses and then finding out the efficiency of the machine by various methods. The methods are broadly classified as:

1. Direct 2. Indirect and 3. Regenerative methods of testing

1. Direct method of testing : In this method the DC machine is actually loaded to the required extent, the Input and output are measured and then the **η** is calculated as **η = Output/Input**
    1. This method is generally employed only for small motors. The motor is loaded by a friction pulley arrangement.(braking)
    2. The main drawback of this method is accuracy of output power measurement is limited.
    3. Difficult to provide braking load for a large capacity motor.
2. Indirect method of testing: In this method the machine is not subjected to full load. First on no load the constant losses are measured and then efficiency is estimated at various loads. Swinburne's test and Hopkinson's test come under this category. Only shunt motors can be tested using these methods. Series motors cannot be tested with this method since they cannot be run on no load.
3. Regenerative method of testing: In this method a motor generator is pair is used which are powered by each other. Thus only losses are drawn from the mains power supply. Hopkinson's test comes in this category.

**Brake Test:** This is a direct method of testing. In this method the motor is put on a direct friction load arrangement with a belt and a pulley as shown in the figure below.

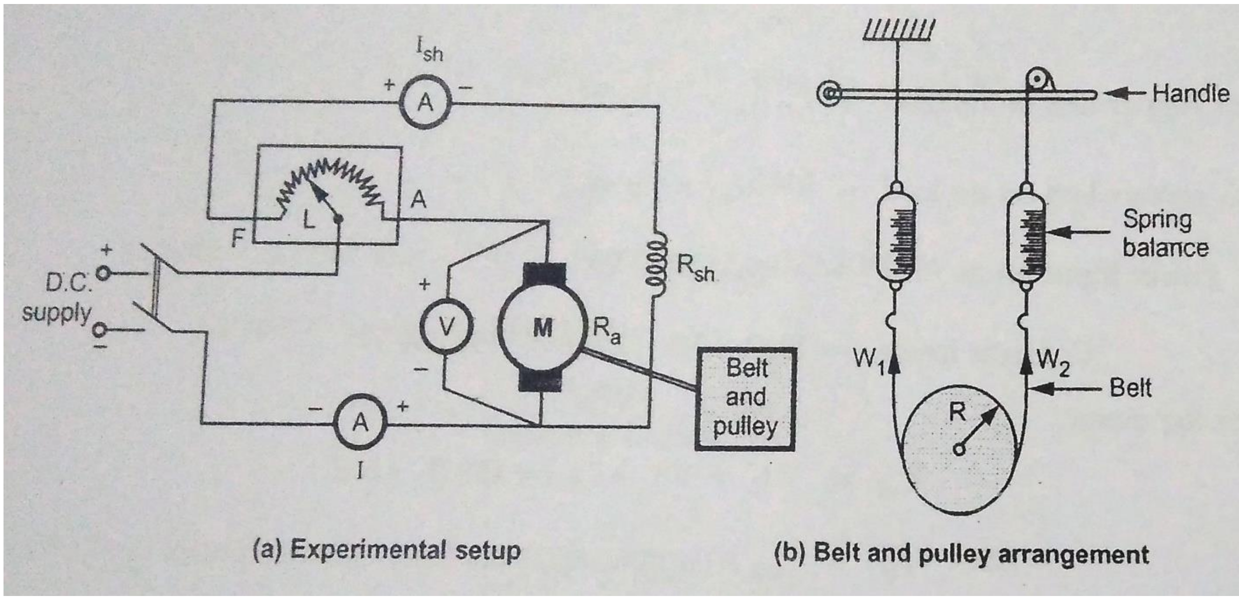



**Figure: Brake Test setup**

By adjusting the tension in the pulley the motor can be subjected from no load to its full load capability. Since the load is applied by the physical braking action, the test is called the ***Brake test***.

The tension in the belt is adjusted by using the handle. The tension (kgf) is obtained from the spring balance readings. The net force applied on the pulley by this braking arrangement is given by:

**Net force = $(W_1 - W_2)$ Kgf = $9.81(W_1 - W_2)$ Nw**

Where R = Radius of the pulley in meters

N = Speed in RPM

$W_1$ and $W_2$ = Spring balance readings on the tight side and on the slack side of the Pulley respectively.

With this force exerted on the pulley, the load torque applied on the motor shaft is given by:

$$\tau_{load} = \text{Net force x Radius of the pulley} = 9.81(W_1 - W_2)R \text{ Nw.m}$$

With this applied load torque $\tau_{load}$, the output power(mechanical ) of the motor is given by:

$$P_{out} = \tau_{load} \times \omega = \tau_{load} \times 2\pi N/60 \text{ W}$$

The input power (electrical) to the motor is given by : $P_{in} = VI$

**Thus we have $\eta = P_{out}/P_{in} = [\tau_{load} x 2\pi N/60]/VI$**

Apart from the efficiency, we can also find out all the characteristics like ***Torque vs Speed*** , ***Speed vs Armature current*** and ***Torque vs armature current*** of the motor by noting down the currents and voltage along with the speed N at various load settings. The speed is measured by using physical contact type Tachometer.

*Advantages:*

1. Efficiency can be found out in the actual working conditions.
2. The method is simple and easy to perform.
3. The test can be performed on any type of DC machine.

*Disadvantages:*

1. Due to the friction lot of energy is wasted in the form of heat. Hence the test is quite expensive and is suitable for only small machines.
2. Since heat energy is not accounted for, the efficiency observed would be inaccurate to that extent.

**Swinburne's test:**

This is a test to determine the efficiency of any DC Machine (Motor or Generator) without conducting the actual test at the required load. The test is conducted just at no load and the constant losses are found out when the machine is running as a motor. Then the efficiency is found out by calculating the variable losses at the required load. This method is formulated by Sir James Swinburne and hence it is called Swinburne's test. This comes under the category *indirect method of testing*. The test setup required to conduct this test is shown in the figure below.

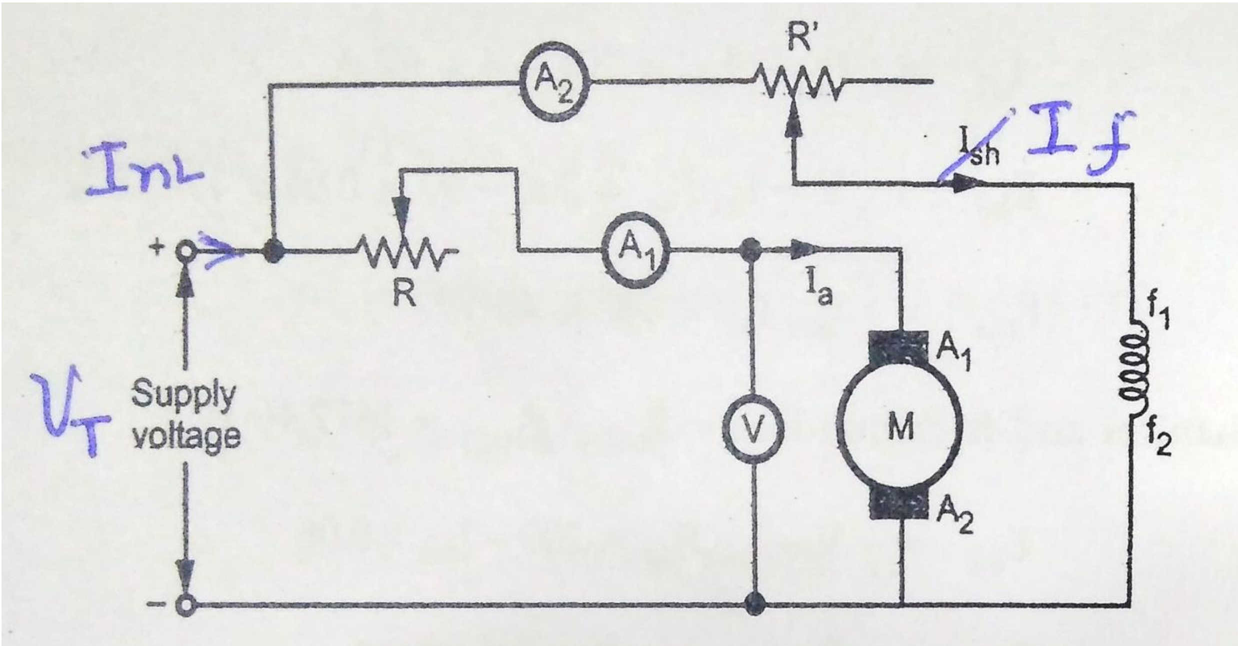



**Figure: Swinburne's Test Setup**

***The machine is run as a motor*** on no load at normal terminal voltage $\mathbf{V_T}$ , at normal speed and the armature current $\mathbf{I_A}$ & field current $\mathbf{I_F}$**(**$\mathbf{I_{sh}}$ in figure ) are measured.

- Then the no load armature current $\mathbf{I_{NL} = I_A + I_F}$
- Variable losses on no load = $\mathbf{I_A^2 .R_A}$ (Machine's armature resistance can be measured directly and these losses can be calculated)
- Input to the motor = $\mathbf{V_T. I_{NL}}$ **= Total losses** (Since the machine is on no load there is no output. i.e. the entire input power on no load goes as losses.)
- Therefore constant losses $\mathbf{P_C}$ **= (Total losses - Variable losses) =** $\mathbf{(V_T. I_{NL})-( I_A^2 .R_A)}$

***Using these constant losses*** $\mathbf{P_C}$***, the efficiency of the machine can be estimated at any other load when working either as a Motor or as a Generator.***

*Working as a Generator delivering a load current of* $\mathbf{I_L}$ *amperes at a terminal voltage of* $\mathbf{V_T}$ *volts:*

Power output = $\mathbf{V_T. I_L}$

Armature current $\mathbf{I_A = I_L + I_F}$ ($\mathbf{I_F}$ is same as obtained in the No load test )

Variable loss = $\mathbf{I_A^2 R_A}$ ($R_A$ is obtained from the no load test or from Machine data)

**Efficiency = (output/Input) = [output/(output + Total losses)] =** $\mathbf{(V_T. I_L)/( V_T. I_L+ I_A^2R_A+P_C)}$

**(**$P_C$ is obtained from the No load test and $I_A^2R_A$ is calculated using $I_A$ corresponding to the required $I_L$ at which the efficiency is to be calculated**)**

*Working as a Motor drawing a load current of* $I_L$ *amperes from a supply terminal voltage of* $V_T$ *volts:*

- Power in put = $V_T.\ I_L$

Armature current $I_A = I_L - I_F$ ($I_F$ is same as obtained in the No load test )

Variable loss = $I_A^2\ R_A$ ($R_A$ is obtained from the no load test or from Machine data)

**Efficiency = (output/Input) = [(Input-Total losses)/ Input] =** $[V_T.\ I_L-(I_A^2R_A+P_C)]/(V_T.\ I_L)$

**(**$P_C$ is calculated and obtained from the No load test and $I_A^2R_A$ is calculated using $I_A$ corresponding to the required $I_L$ at which the efficiency is to be calculated)

*Advantages of Swinburne's test*:

- This is a very simple to determine the efficiency of the machine at any load just by conducting the no load test.
- The power required is very less compared to the direct full load test.

*Disadvantages of Swinburne's test*:

- This test can be done on Shunt machines only.
- The speed and flux are assumed constant. But the speed will fall with loading. Fall in speed results in lesser friction and windage losses. Change in flux will change the core losses.
- The temperature of the machine changes while running on load. Hence the assumption that $R_A$ is same as that of the No load test is not correct.
- These reasons contribute to the difference in the efficiency obtained from the Swinburne's test and actual load test.

**Hopkinson's test:** In this set up two identical DC machines are coupled mechanically and tested together. One of the machines works as a motor and drives the other machine which works as generator and it's electrical output in turn is connected back to the motor. Hence this comes under the regenerative category and is also called a Back to Back test. The motor is connected to the mains supply and it draws power from the mains only to compensate for the losses in the two machines since the major power required by each machine is derived from the other machine. Since the power consumption is only to the extent of the losses they can be tested up to full load.

Figure below shows the Hopkinson's test up with all the measured voltage and current parameters marked clearly.

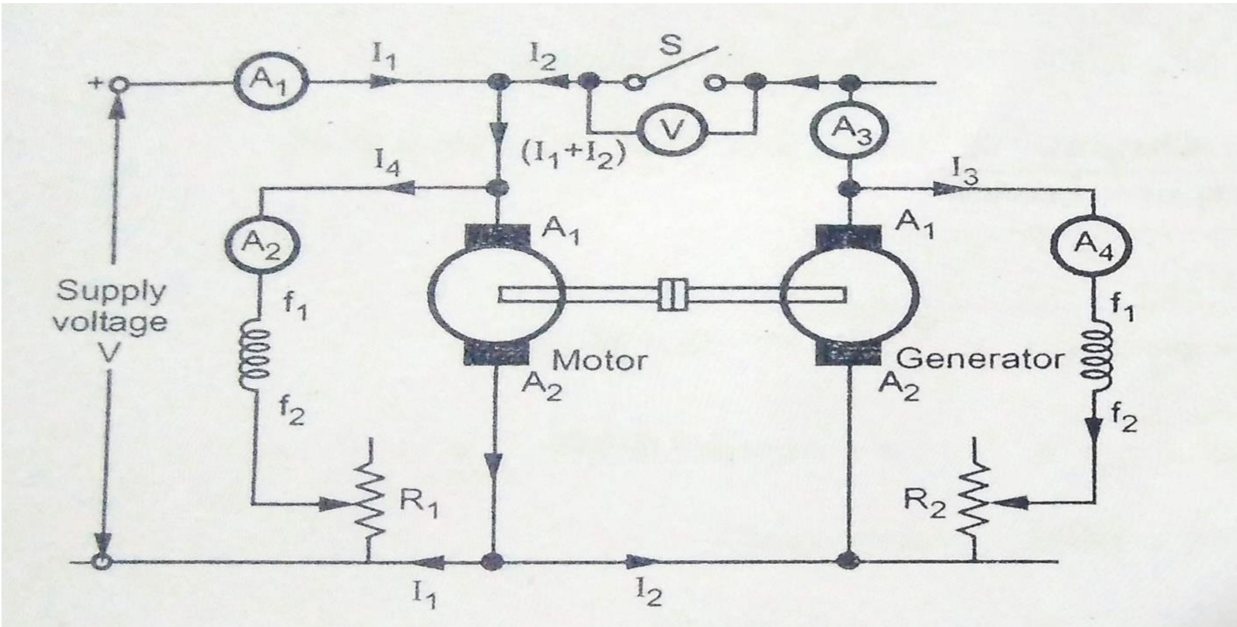



**Figure: Hopkinson's Test Setup**

**(Ammeters are to be redesignated as 'A2 to A4' , 'A3 to A2 ' and 'A4 to A3' so that the current designations will match )**

Initially the switch **S** is kept open and the Motor is run and brought to the rated speed by adjusting the field current using the field rheostat $\mathbf{R_1}$ . The Generator Voltage is adjusted by adjusting it's field rheostat $\mathbf{R_2}$ until the voltmeter reads zero volts. (This indicates that the Generator voltage is same as that of the Motor both in amplitude and polarity) This prevents flow of any high circulating currents when the switch **S** is closed and the two machines are connected back to back.

Now the switch is closed thus bringing the machines back to back and the load on both the machines can be increased gradually by increasing the Generator excitation or decreasing the Motor excitation. The readings from all the meters are taken at every load setting for further evaluation. Let us spell out clearly all the parameters for ease of further derivations.

**V = Terminal Voltage (Supply Voltage)**

$\mathbf{I_1}$ **= Current drawn from mains supply**

$\mathbf{I_2}$ **= Current supplied by the generator to the motor**

**$I_3$ = Excitation Current of Generator**

**$I_4$ = Excitation Current of Motor**

**$R_a$ = Armature resistance of each machine**

**1. Equal efficiency : Let us now first find out the Efficiency 'η' assuming it to be same for both the machines:**

**Input to the motor = $V (I_1 + I_2)$**

**Output of the motor = η x Input to the motor = $\eta \times V (I_1 + I_2)$**

**This output of the motor is given as input to the generator. Hence**

**Input to the Generator = $\eta \times V (I_1 + I_2)$**

**Output of the generator = η x Input to the generator = $\eta \times \eta \times V (I_1 + I_2) = \eta^2 \times V (I_1 + I_2)$**

**But the output of the generator can also be given as = $V I_2$ and equating these two we get**

**$\eta^2 \times V (I_1 + I_2) = V I_2$ from which we get**

$$\eta = \sqrt{[ I_2 / (I_1 + I_2) ]}$$

**2. Un equal efficiency: Let us now find out the Efficiency 'η' assuming it to be unequal for the two machines:**

In this analysis, the stray losses (Constant Losses) are assumed to be same for both the machines where as the field and armature copper losses are different (Since when the efficiencies are different it means that the currents are not the same and hence the copper losses also will not be same)

*So first let us determine the Copper losses of the two machines independently and then the constant losses of both machines together can taken as the difference between the input power from the main supply and the total copper losses.*

Armature copper losses in Generator = $(I_2 + I_3)^2 \times Ra$

Armature copper losses in Motor = $(I_1+I_2 - I_4)^2 \times Ra$

Field copper losses in Generator = $V I_3$

Field copper losses in Motor = $V I_4$

We know that the total losses in both the machines put together are equal to the input power from the mains supply i.e **$V I_1$**

Hence we can take that the total stray losses for both the machines put together are the difference between the input power and all the copper losses put together. Hence

Stray losses of both machines together = **$V I_1 – [(I_2 + I_3)^2 \times Ra + (I_1+I_2 – I_4)^2 \times Ra + V I_3 + V I_4]$** = Say **$W_s$**
and stray losses of each machine = **$W_s/2$**

Now that we know the variable losses (Armature and Field Copper losses) and the constant (stray) losses for both the machines we can easily find out the efficiencies of both Generator and Motor using the above data as shown below.

**Efficiency of Generator:**

**Total losses = Generator's variable losses + Stray losses of one machine**

$$= [(I_2 + I_3)^2 \times Ra + V I_3 + W_s/2]$$

**Output of Generator = $V I_2$**

**Efficiency of Generator $\eta_G$ = Output/ Input = Output/ ( Output + Losses)**

$$= V I_2 / V I_2 + [(I_2 + I_3)^2 \times Ra + V I_3 + W_s/2]$$

**Efficiency of Motor:**

**Total losses = Motor's variable losses + Stray losses of one machine**

$$= [(I_1 + I_2 – I_4)^2 \times Ra + V I_4 + W_s/2]$$

**Input to Motor = $V (I_1 + I_2)$**

**Efficiency of Motor $\eta_M$ = Output/ Input =( Input – losses ) / Input**

$$= [V (I_1 + I_2) - [(I_1 + I_2 – I_4)^2 \times Ra + V I_4 + W_s/2]] / V (I_1 + I_2)$$

## Field's test:

**Introduction and Methodology:**

This test is for finding out the losses and efficiency of DC Series Motors by direct testing, since series motors cannot be tested on no load. The test is named after the inventor of test method 'MB Field '. Series motors which are normally used for Traction are available as pairs and hence this test is devised on two motors which are coupled mechanically. The test setup is shown in the figure below. One machine works as motor supplying power to the other one working as Generator. Their mechanical and iron (core) losses (put together called as *stray losses* or *constant losses*) are made equal by: (i) running them with equal speed and (ii) by connecting their both field windings in the motor armature circuit to the Motor input supply such that both the machines are equally excited . The load resistance is adjusted till the Motor draws the rated current as read by ammeter A1. In this condition all the other parameters are noted down from the respective meter readings as per the nomenclature given below.

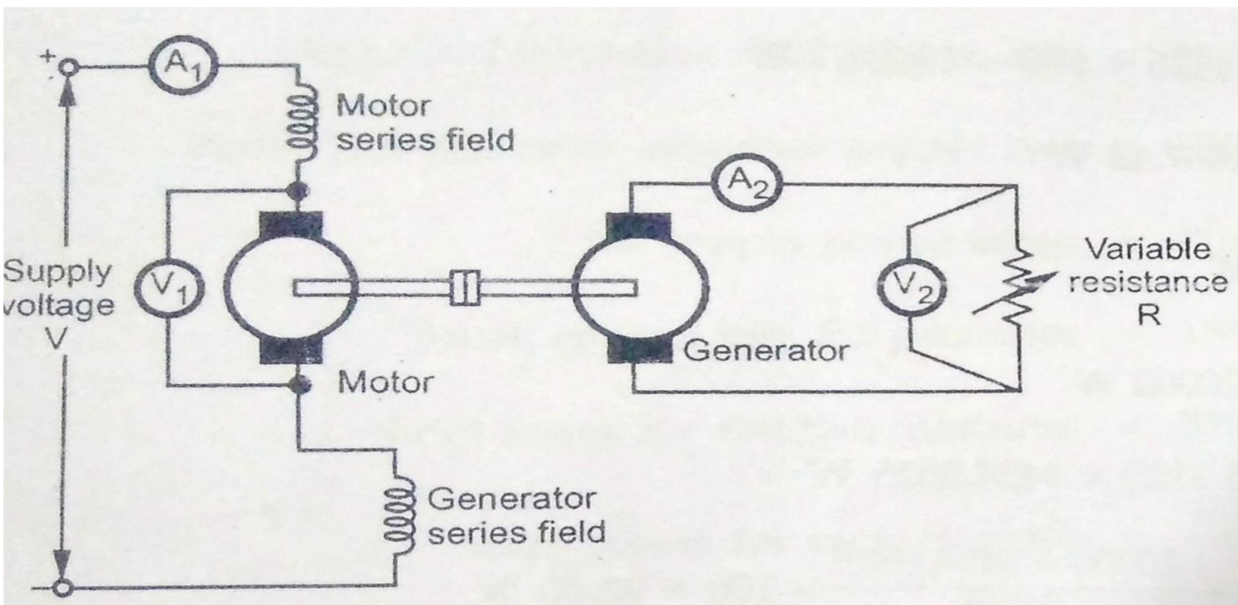



**Figure: Field's Test Setup**

**Nomenclature:**

V = Supply voltage

$V_1$ = Motor Supply voltage

$V_2$= Generator output voltage connected across the variable load resistance R

$I_1$ = Motor armature current and also the field current of both Motor and Generator

$I_2$ = Generator armature current

Also Let **$R_A$** = Motor armature resistance and **$R_F$** = Motor field resistance ( which can be measured independently or can be taken from the machine data )

***From this data obtained in this full load condition, the stray losses, copper losses and then efficiency of both the Motor and the Generator can be found out as below:***

**Stray Losses:**

Input to the total Motor Generator test setup**: $V I_1$**

Output of Generator = **$V_2 I_2$**

Total losses of both motor and generator = **$W_T = V I_1 - V_2 I_2$**

But total losses of both motor and generator **$W_T$** are also equal to (Armature Copper losses + Field Copper losses + Stray losses ).Thus

$$W_T = V I_1 - V_2 I_2 = (I_1^2 + I_2^2) R_A + 2I_1^2 R_F + W_S$$

And total Stray Losses = Total Losses – Total Armature and Field copper losses

$$= VI_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F ] \text{ and}$$

Stray losses per machine = Total Stray Losses/2 = **$W_S = [V I_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F ]] / 2$**

***Now using this value of stray losses of each machine, the efficiency of the machine as a Motor and as a Generator can be found out as below.***

**η as a Motor:**

Input to Motor = **$V_1 I_1$**

Total losses = Armature and Field Copper losses + Stray losses = **$I_1^2(R_A + R_F) + W_S$**

Output of Motor = Input – Losses = **$V_1 I_1 - \{I_1^2(R_A + R_F) + W_S\}$**

$$\textbf{Efficiency of the motor} = \eta_M = \text{Output/Input} = [V_1 I_1 - \{I_1^2(R_A + R_F) + W_S\}] / V_1 I_1$$

**η as a Generator:**

*This not very important because the machine is working in separately excited condition. However just for completion sake let us find it out.*

Output of Generator = $\mathbf{V_2 I_2}$

Total losses = Armature and Field Copper losses + Stray losses = $\mathbf{I_2^2R_A + I_1^2R_F + W_S}$

Input to the Generator = Output + Losses = $\mathbf{V_2 I_2 + I_2^2R_A + I_1^2R_F + W_S}$

**Efficiency of the Generator = $\eta_G$ = Output/Input = $V_2 I_2 / (V_2 I_2 + I_2^2R_A + I_1^2R_F + W_S)$**

**Retardation test or Running down test:** This is an indirect test similar to the Swinburne's test where in the constant (Stray losses) losses are first determined and then the efficiency at any load when the machine is working both as a Generator and Motor are estimated on the same lines. However the constant losses in this test are determined by a different principle i.e. by finding out the kinetic energy spent by a rotating mass during the process of retardation from the rated speed to zero speed and then calculating the rate of change of kinetic energy which is equal to the Power Loss.

The setup to conduct this test is shown in the figure below.

*Test Procedure*: The motor is started and taken to a speed higher than the rated speed of the machine. Then the supply to the motor is cutoff by moving the Two Pole Two way switch from the supply side to the open terminals. The armature then slows down with its' own inertia and its stored energy is used up to supply the constant rotational losses (stray losses) like iron, friction, windage etc. This power loss is found out from the following principle.

If **'I'** is the moment of Inertia of the Armature, and **'ω'** is the angular velocity, then the kinetic energy of the armature is given by:

$$KE = \tfrac{1}{2} I\omega^2 \text{ and}$$

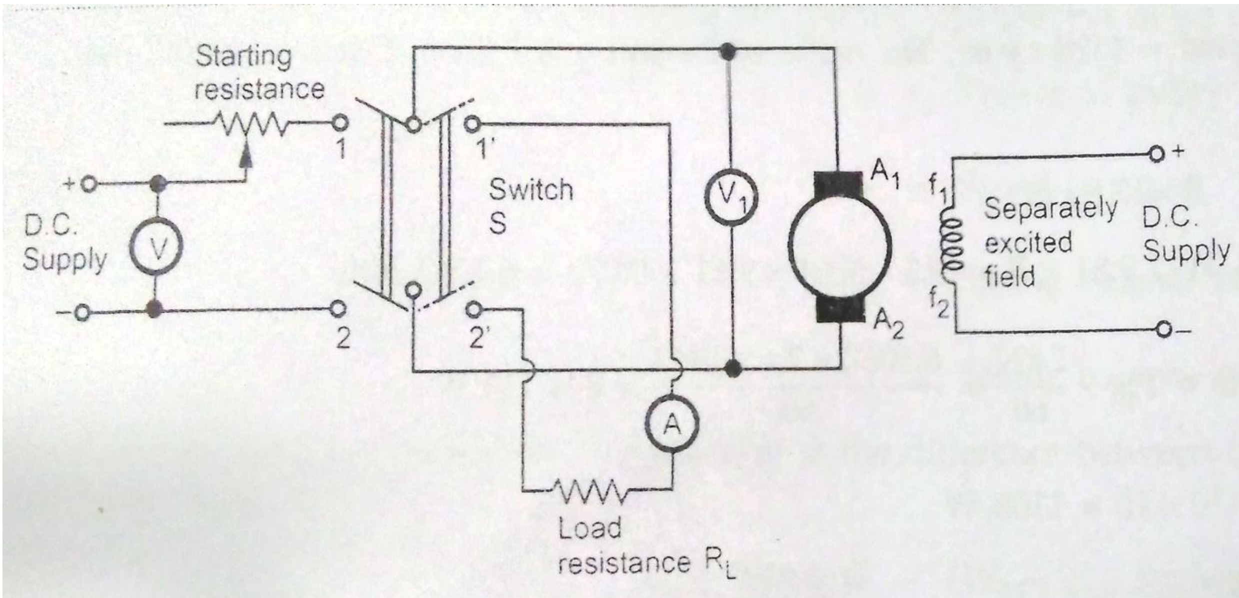



**Figure: Retardation test setup**

The power loss which is the rate of change of Energy is given by**:**

$$P_L = d/dt\ (\tfrac{1}{2}\ I\omega^2) = I .\ \omega\ .\ d\omega/dt$$

Substituting the value of **'ω'** in terms of the speed in RPM **'N'** ( 2πN/60) we get

$$P_L = (2\pi/60)^2\ I .\ N\ .\ dN/dt$$

So, to find out the stray losses we must know **'I'**, the moment of inertia and **dN/dt**, the rate of change in speed. The method of finding out these quantities is given below.

*The method of finding out* ***dN/dt :***

When the motor is cutoff from the input supply, the speed starts falling down. The motor back e.m.f. as read by the Voltmeter $V_1$ connected across the motor is noted down as a function of time. Since we know that the back e.m.f is proportional to the speed we can convert the $V_1$ reading into speed in RPM and plot it as a function of time as shown in the figure below. In this figure, at the point **C** corresponding the rated speed a tangent AB is drawn whose slope gives the rate of change in speed dN/dt.

Thus: **dN/dt = OA(RPM)/OB(Time)**

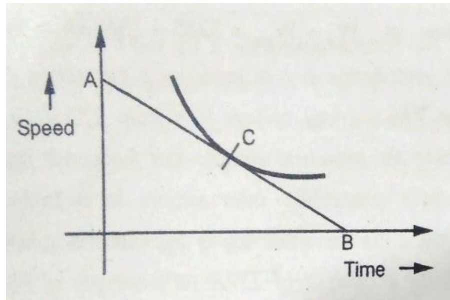



**Figure: Speed fall during the retardation test**

***Determination of Moment of Inertia (I):***

**Method -1: Using Flywheel**

The same test as earlier is repeated after adding a *flywheel* of known Moment of Inertia **'$I_1$'** to the armature shaft of the motor and the resulting rate of change of speed is obtained. Let us call the earlier rate of change as $dN/dt_1$ and the second rate of change with added inertia as $dN/dt_2$.

Then, since the losses can be assumed to be same with or without the new flywheel we have the following relations:

In the first case without flywheel: $P_L = (2\pi/60)^2\ I.\ N\ .\ dN/dt_1$

In the second case with added flywheel: $P_L = (2\pi/60)^2\ (I+ I_1).\ N\ .\ dN/dt$

After equating the two equations and simplifying we get : $I = I_1 (t_1) /( t_2 - t_1)$

**Method -2: Without using a Flywheel (Using Resistance Braking)**

- First, the switch is taken from supply side to the open condition and then the time taken ( say $t_1$) for the motor to slow down from a speed slightly higher than the rated speed to a speed slightly lower than the rated speed (say $\delta N$) is noted down with just the Armature alone like in the earlier method step -1 (without any added external inertia)
- Then again the switch S is moved from the supply position to the Resistance position quickly and again the time taken(say $t_2$) for the same change in speed (same $\delta N$)is noted down. By this effectively we are connecting an electrical load across the armature in which the stored electrical power is dissipated thus providing an additional retarding torque.
- This additional power loss due to the resistance is given by the product of the Average Voltage across the Armature (say V ) and the average current (say $I_A$ ) that flows into the Braking Resistance R i.e. $I_A^2 ( R_A +R )x V$ = say **W'**
- Then the powers dissipated during the above two steps are given by

1. **W = $(2\pi/60)^2$ I. N . $dN/dt_1$ (Just due to the armature Inertia)**
2. **W + W' = $(2\pi/60)^2$ I. N . $dN/dt_2$ (Due to the armature Inertia and the braking resistance )**

## Separation of Constant losses:

The theory required for the purpose of Separation of Constant losses is explained below.

At any given excitation:

- Friction losses and hysteresis losses are both proportional to speed .
- Windage losses and eddy current losses are both proportional to square of speed.
- Hence Friction losses = **AN** Watts, Windage losses = **$BN^2$** Watts, Hysteresis losses = **CN** Watts, and Eddy current losses = **$DN^2$** Watts where **N** = speed and **A,B,C** and **D** are constant coefficients
- Further the coefficient **C** of hysteresis losses is proportional to **$B_{max}^{1.6}$** and the coefficient **D** of Eddy current losses is proportional to **$B_{max}^2$**

*The other standard relation: For a motor on no load, power input to the armature is the sum of the armature copper losses and the above losses.*

Hence from a no load test we can get the constant losses as usual and then equate them to the constant losses with the above categorization as shown below:

Power input to the armature = **$V.I_a$** watts.

Armature copper losses = **$I_a^2.R_a$** watts

Constant Losses = **W** = **$V.I_a - I_a^2.R_a = (A + C)N + (B + D)N^2$**

$$W/N = (A+C) + (B+D)N.$$

First the test is conducted with rated field current as per the following procedure:

1. The motor is started on no load with field current set to rated value by adjusting the field auto transformer.
2. The armature voltage is increased till the speed is about 200 rpm more than the rated value.
3. Now, the speed is gradually decreased by decreasing the armature voltage, the values of armature voltage, armature current and speed are noted down.
4. From this data **W** is calculated as explained above and the graph between **W/N & N** is plotted as shown in the figure below.

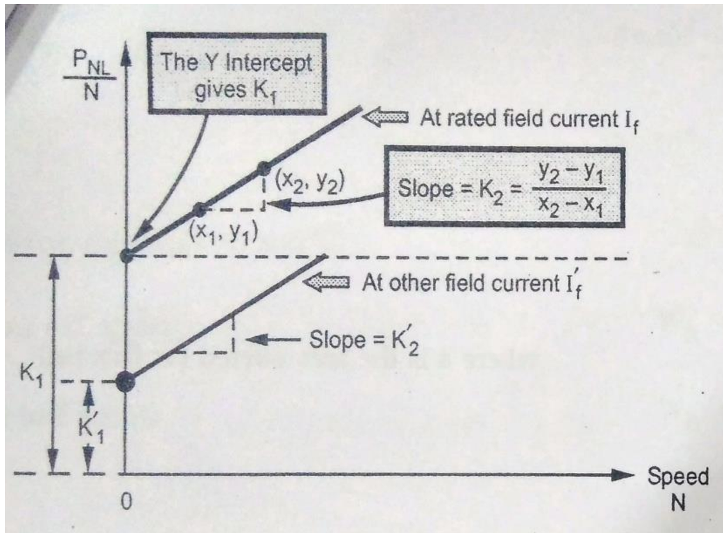



**Figure : Plot of $P_{NL}$ / N vs. N**

5. The graph between **$P_{NL}$ /N & N** is a straight line from which **(A+C)** and **(B+D)** can be found. **(A+C)** is the **y** axis intercept and **(B+D)** is the slope of the straight line.
6. In order to separate out **A, B, C and D**, the test is repeated again with reduced field current and the graph between **$P_{NL}$ /N & N** is drawn again as shown in the figure which is a straight line given by **$P_{NL}$ /N = (A+C') N + (B+D') N** from which we can again find out **(A+C')** and **(B+D').**

   At the reduced excitation, friction and windage losses are still **AN** and **$BN^2$**, but hysteresis losses become **C'N** and eddy current losses become **$D' N^2$**. We can now obtain **(A+C')** and **(B+D')** as before from the second straight line .

As already indicated the coefficient of hysteresis loss **C** is proportional to **$B_{max}^{1.6}$**, and the coefficient of eddy current loss D is proportional to **$B_{max}^2$**. Since the core is common we can replace maximum flux density **$B_{max}$** with maximum flux density **$Ø_{max}$** in the core and then the ratio of fluxes corresponding to the normal and reduced excitation would become :

**(C/C')** = **$(Ø_{max} / Ø'_{max})^{1.6}$** also equal to **$(E_b/E'_b)^{1.6}$** since Back e.m.f is proportional to **$Ø_{max}$**

**(D/D')** = **$(Ø_{max} / Ø'_{max})^2$** also equal to **$(E_b/E'_b)^2$** since Back e.m.f is proportional to **$Ø_{max}$**

During the two tests we can get the values of back e.m.f s from the measured values of armature supply voltage **V** , armature current **$I_a$** and armature resistance **$R_a$** using the relation: **$E_b = V - I_a.R_a$** and thus get the values of **(C/C')** and **(D/D') .** We can also get the values of **(C'-C)** and **(D'-D)** by subtracting **(A+C)** from **(A+C')** and **(B+D)** from **(B+D')**

From these values we can get all the four coefficients **A,B,C** and **D** and thus separate the constant losses **$P_{NL}$** into ***Friction***, ***Windage***, ***Hysteresis*** and ***Eddy current*** losses

Since the change in speed δN is same in both the cases, dividing the expression 2 by expression 1 we get

**$(W + W')/W = (1/dt_2) / (1/dt_1) = dt_1/dt_2 = t_1/t_2$** and after simplification we get

**$W = W' [t_1/(t_1 - t_2)]$ which is equal to the stray losses ( Mechanical plus Core losses )**

## Important concepts and Formulae:

- **The efficiency of a DC Machine is defined as $\eta = (P_{out}/P_{in}). 100$ %**

  ***Efficiency of Generator:***
  **$\eta = (P_{out}/P_{in}). 100 \% = (V_T.I_L)/(V_T.I_L + I_A^2 R_A + I_F^2 R_F + W_C).100\%$**

  ***Efficiency of Motor:***
  **$\eta = (P_{out}/P_{in}). 100 \% = [\{V_T.I_L - (I_A^2 R_A + I_F^2 R_F + W_C)\}/(V_T.I_L)] . 100\%$**

- **The condition for maximum efficiency :**
- **Constant losses $P_C$ = Variable losses ( $I_L^2R_A$ ) or $I_L = \sqrt{(P_C/R_A)}$**

## Illustrative examples:

**Ex.1:** A DC shunt motor having a full load efficiency (**η** )of 85 % takes a line current of 27A from 250 Volts mains on full load. If $R_a = 0.5\Omega$ and $R_f = 125\ \Omega$, find the constant losses, load current for maximum efficiency and the maximum efficiency.

**Solution:**

Input power at full load = Full load current x Rated voltage = 250 x 27 = 6750 W

Output power = Input power x η = 6750 x 0.85 ( η = 85% ) = 5737.5 W

Hence Total losses = Input power – Output power = 6750 – 5737.5 = 1012.5 W

We know that Total losses = Variable losses ( $I_a^2R_a$ ) + constant losses.

$I_f$ = Rated Terminal Voltage/ Field resistance = 250 / 125 = 2 A

For the Shunt motor armature current $I_a = I_l - I_f = 27 - 2 = 25$ A

Variable losses = $I_a^2 R_a = 25^2 \times 0.5 = 312.5$

Constant losses = Total losses − Variable losses = 1012.5 − 312.5 = 700 W

We know that the condition for maximum efficiency is: **Variable losses = Constant losses**

i.e. $I_a^2 R_a$ at maximum efficiency = 700 ∴ $I_{a\,@max.\,\eta} = \sqrt{700 / 0.5} = \sqrt{1400} = 37.42$ A

∴The load current at maximum η : $I_{l@max.\,\eta} = I_{a@max.\,\eta} + I_f = 37.42 + 2 = 39.42$

Input power at maximum η = $I_{l\,@max.\,\eta}$ x Rated terminal voltage = 39.42 x 250 = 9855 W

O/P power at maximum η = I/P power at maximum η−Total losses = 9855−(700 +700) = 8455 W

(Since variable losses = constant losses = 700)

Maximum efficiency = Out power at maximum efficiency/ Input power at maximum efficiency

= 8455 / 9855 = 0.858 or 85.8 %

**Ex.2:** A 100 Kw 500 V DC shunt machine when run as a motor on no load at rated speed and voltage takes a line current of 10 A and a shunt field current of 2.5 A . Resistance of the armature is 0.15 Ω. Estimate the efficiency of the DC machine when running as a generator (a) at full load (b) at half full load.

*First the constant losses of the Machine are obtained from the data we have when the machine is run as a motor on no load at rated speed and voltage*:

Input power on no load = Rated voltage x Input current on no load = 500 x 10 = 5000 W

Field current $I_f$ = 2.5 A

No load Armature current $I_a = I_{l\ no\ load} - I_f = (10 - 2.5) = 7.5$ A

Variable loss at no load = $(I_{a\ on\ no\ load})^2 \times R_a = 7.5^2 \times 0.15 = 8.4375$ W

**Constant Losses** = ( Input power– Variable losses)(on no load) = 5000 - 8.4375 = **4991.56** W

*Next we will calculate the efficiency in different conditions:*

*(a) Running as a generator at full load:*

Full load output (line) current = 100 x 1000 / 500 = 200 A

Full load armature current = Full load line current + Field current = 200 + 2.5 = 202.5 A

Variable (Armature copper) losses on full load = $I_a^2 R_a$ = $202.5^2$ x 0.15 = 6150.94 W

Total losses @ full load = Constant Losses + Variable losses on full load = 4991.56 + 6150.94

= 11142.5 W

**Efficiency at full load (Working as Generator)** = Output / Input = Output / Output + Total losses @ full load = 100000 / 100000 + 11142.5 = 0.8997 or **89.97 %**

*(a) Running as a generator at half full load:*

Half Full load output (line) current = 50 x 1000 / 500 = 100 A

Half Full load armature current = Half Full load line current + Field current = 100 + 2.5 = 102.5 A

Variable (Armature copper) losses on half full load = $I_a^2 R_a$ = $102.5^2$ x 0.15 = 1575.94 W

Total losses @ half load = Constant Losses + Variable losses on half load = 4991.56 + 1575.94

= 6567.5 W

**Efficiency at half full load (Working as Generator)** = Output / Input = Output / Output + Total losses @ full load = 50000 / 50000 + 6567.5 = 0.8839 or **88.39 %**

**Ex.3**: A 500 V DC shunt machine takes 5A when running light (on no load) at rated speed and rated voltage as a motor. Calculate the out output power and efficiency when the machine is run as a Motor and taking an Input current of 80 A. Calculate the line current at which the efficiency is maximum and the value of maximum efficiency. $R_a$ = 0.2 Ω and $R_f$ = 250 Ω

***First the constant losses of the Machine are obtained from the data we have when the machine is run as a motor on no load at rated speed and voltage*:**

Input power on no load = Rated voltage x Input current on no load = 500 x 5 = 2500 W

Field current $I_f$ = Rated voltage / $R_f$ = 500 / 250 = 2 A

No load Armature current $I_a = I_{l\ no\ load} - I_f$ = ( 5 – 2 ) = 3 A

Variable loss at no load = $(I_{a\ on\ no\ load})^2$ x $R_a$ = $3^2$ x 0.2 = 1.8 W

**Constant Losses** = ( Input power– Variable losses)(on no load) = 2500 – 1.8 = **2498.2** W

***Next we will calculate the output power and efficiency when the Machine is running as a motor and taking an input current of 80 A :***

Armature current = Line current (Input Current) - Field current = 80 - 2 = 78 A

Variable (Armature Copper) losses (with armature current of 78A) = $I_a^2 R_a = 78^2 \times 0.2 = 1216.8$ W

Total losses = Constant Losses + Variable losses at 80 A line current = 2498.2 + 1216.8 = 3715 W

Input Power = 500 x 80 = 40000 W

**Out Put Power at 80 A line current (Working as Motor)** = Input Power – Total losses at 80 A line current = 40000 -3715 = 36285 **W**

**Efficiency at 80 A line current (Working as Motor)** = Output Power /Input Power

= 36285/40000 = 0.9071 or **90.71 %**

***Finally we will calculate the line current at which the efficiency is maximum and the value of maximum efficiency:***

We know that the condition for maximum efficiency is: **Variable losses = Constant losses**

i.e. $I_a^2 R_a$ ( variable Losses ) at maximum efficiency = **2498.2** W

$\therefore \quad I_{a\ @max.\ \eta} = \sqrt{2498.2 / 0.2} = \sqrt{12491} = 111.76$ A

$\therefore$**The line current at maximum η** : $I_{l@max.\ \eta} = I_{a@max.\ \eta} + I_f = 111.76 + 2 =$ **113.76 A**

Input power at maximum η = $I_{l\ @max.\ \eta}$ x Rated terminal voltage = 113.76 x 500 = 56880 W

O/P power at maximum η = I/P power at maximum η—Total losses = 56880 – (2498.2 + 2498.2) = 51883.6 W

(Since variable losses = constant losses = 2498.2)

**Maximum efficiency** = Out power at maximum efficiency/ Input power at maximum efficiency

= 51883.6 /56880 = 0.9122 or **91.22 %**